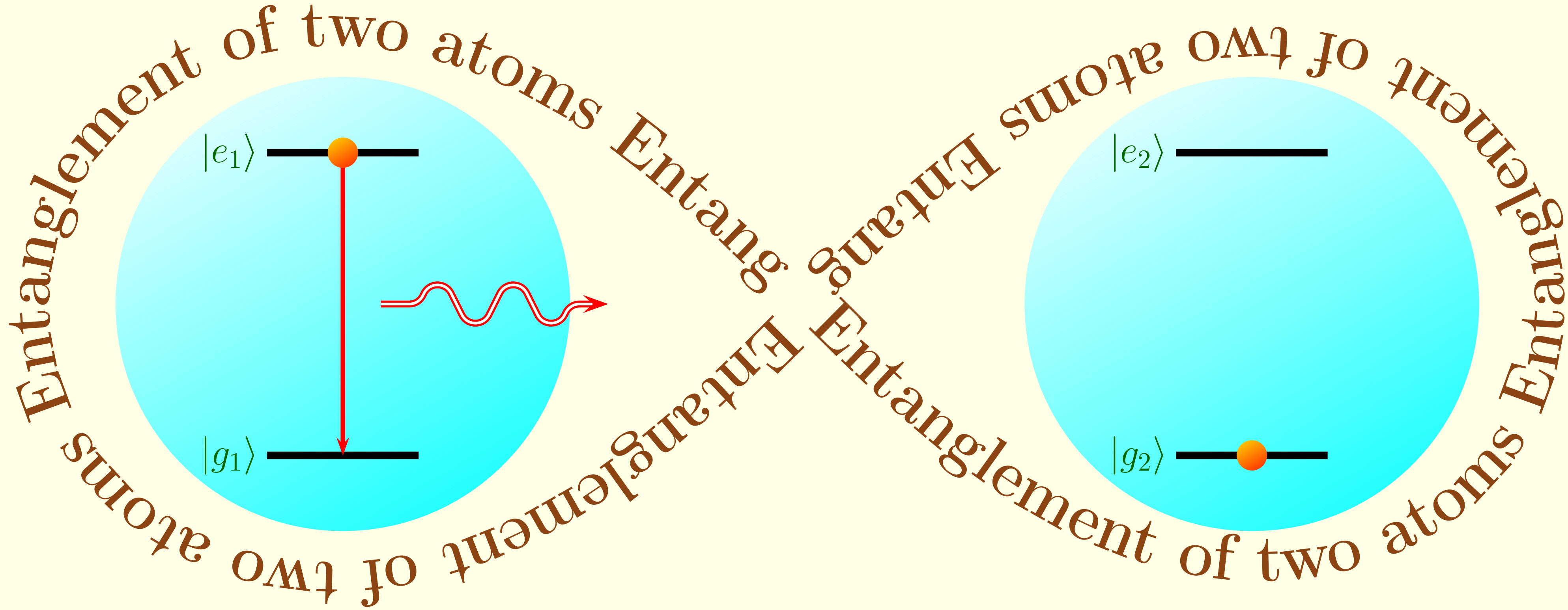
CEWQO 2008



Belgrade, Serbia

Central European Workshop
on Quantum Optics
May 30 – June 3, 2008, Belgrade, Serbia

Dynamics of entanglement in a dissipative environment

Ryszard Tanaś[1] and Zbigniew Ficek[2]

[1] *Adam Mickiewicz University, Poznań, Poland*

[2] *The University of Queensland, Brisbane, Australia*

Outline

Supported by

Polish Ministry of Science and Higher Education

Grant 1 P03B 064 28

The University of Queensland, Australia

Travel grant

1 Density matrix evolution

1.1 Markovian master equation

Evolution of two atoms is described by the equation

$$\frac{\partial \hat{\rho}}{\partial t} = -\,i \sum_{i=1}^{2} \omega_i \left[S_i^z, \hat{\rho}\right] - i \sum_{i \neq j}^{2} \Omega_{ij} \left[S_i^+ S_j^-, \hat{\rho}\right] - \frac{1}{2} \sum_{i,j=1}^{2} \Gamma_{ij} \left(\hat{\rho} S_i^+ S_j^- + S_i^+ S_j^- \hat{\rho} - 2 S_j^- \hat{\rho} S_i^+\right)$$

1 Density matrix evolution

1.1 Markovian master equation

Evolution of two atoms is described by the equation

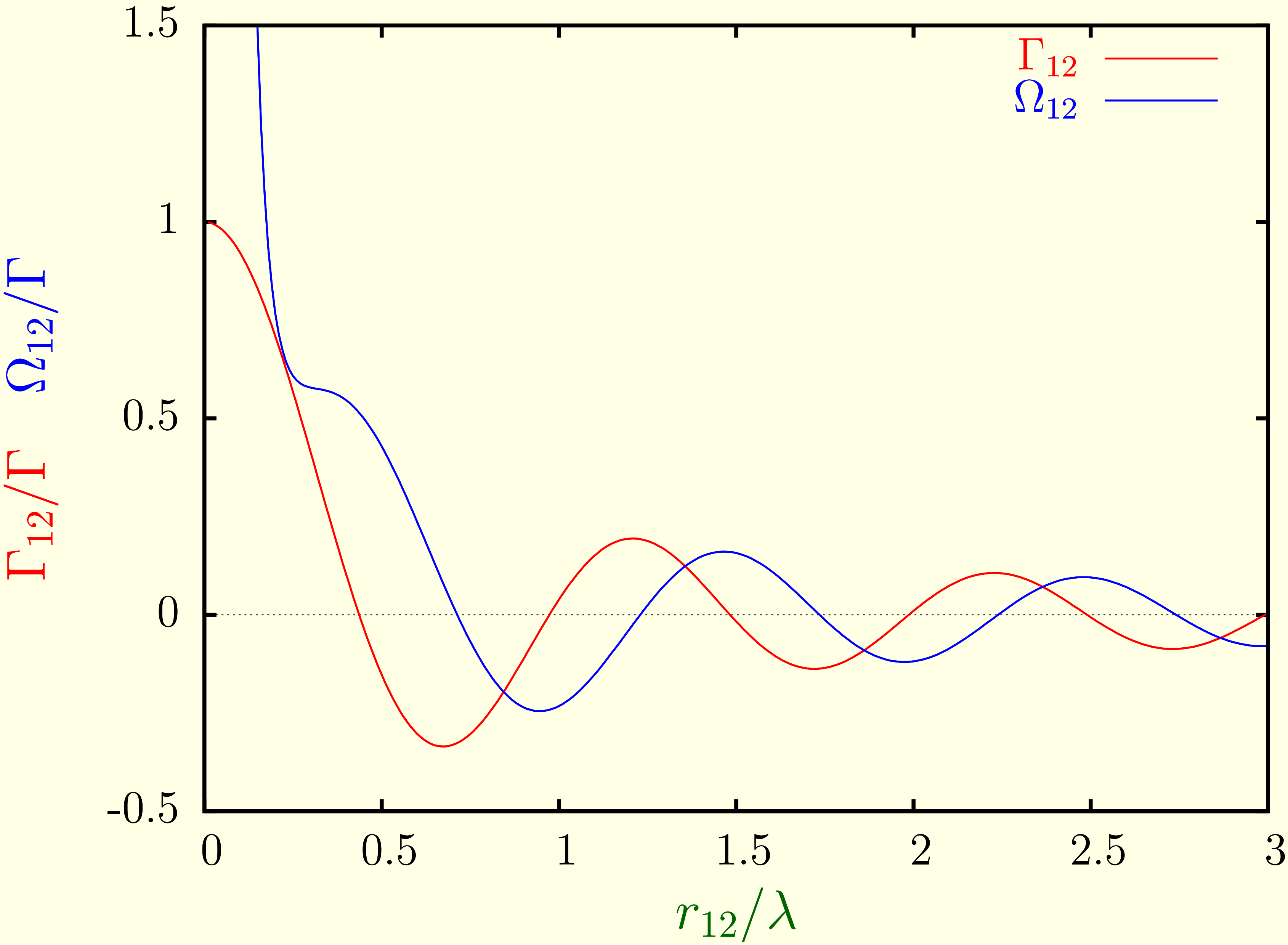
$$\frac{\partial \hat{\rho}}{\partial t} = -i \sum_{i=1}^{2} \omega_i \left[S_i^z, \hat{\rho}\right] - i \sum_{i \neq j}^{2} \Omega_{ij} \left[S_i^+ S_j^-, \hat{\rho}\right]$$

$$-\frac{1}{2} \sum_{i,j=1}^{2} \Gamma_{ij} \left(\hat{\rho} S_i^+ S_j^- + S_i^+ S_j^- \hat{\rho} - 2 S_j^- \hat{\rho} S_i^+\right)$$

Collective parameters:

$\Omega_{12}(r_{12}) = \Omega_{21}(r_{12})$ dipole-dipole interaction

$\Gamma_{12}(r_{12}) = \Gamma_{21}(r_{12})$ collective damping

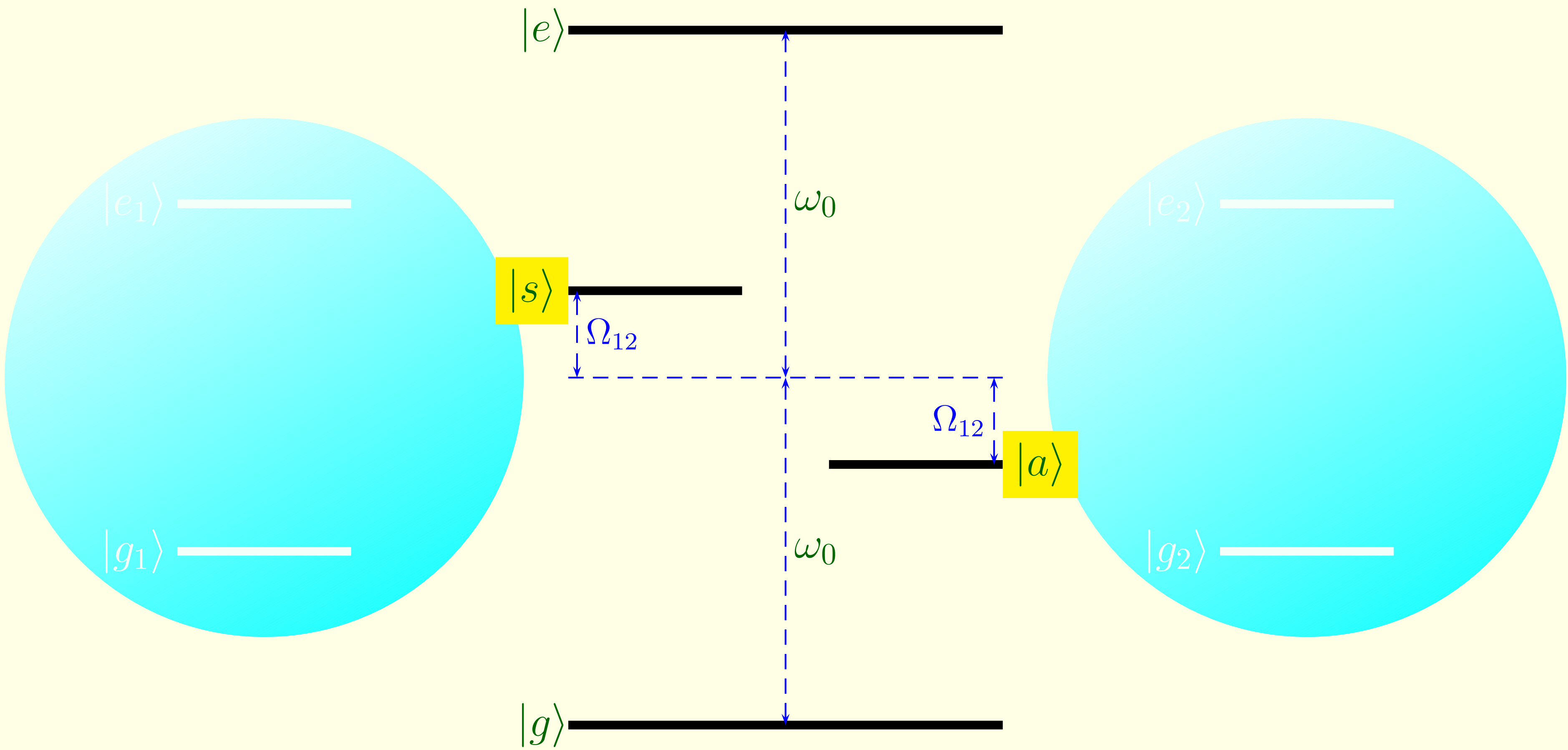


Collective parameters Γ_{12} and Ω_{12} versus r_{12} ($\hat{\boldsymbol{\mu}} \perp \hat{\boldsymbol{r}}_{12}$).

1.2 Collective states

Standard basis — independent atoms:

$\{|1\rangle = |g_1\rangle \otimes |g_2\rangle, \, |2\rangle = |g_1\rangle \otimes |e_2\rangle, \, |3\rangle = |e_1\rangle \otimes |g_2\rangle \, ,$
$|4\rangle = |e_1\rangle \otimes |e_2\rangle\}$



Collective states:

$$\{|g\rangle = |g_1\rangle \otimes |g_2\rangle,\ |e\rangle = |e_1\rangle \otimes |e_2\rangle,$$

$$|s\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle \otimes |g_2\rangle + |g_1\rangle \otimes |e_2\rangle\right),$$

$$|a\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle \otimes |g_2\rangle - |g_1\rangle \otimes |e_2\rangle\right)\}$$

1.3 Standard basis

If the density matrix has the X form initially, . . .

$$\rho(0) = \begin{bmatrix} \rho_{11}(0) & 0 & 0 & \rho_{14}(0) \\ 0 & \rho_{22}(0) & \rho_{23}(0) & 0 \\ 0 & \rho_{32}(0) & \rho_{33}(0) & 0 \\ \rho_{41}(0) & 0 & 0 & \rho_{44}(0) \end{bmatrix}$$

1.3 Standard basis

If the density matrix has the X form initially, . . .

$$\rho(0) = \begin{bmatrix} \rho_{11}(0) & 0 & 0 & \rho_{14}(0) \\ 0 & \rho_{22}(0) & \rho_{23}(0) & 0 \\ 0 & \rho_{32}(0) & \rho_{33}(0) & 0 \\ \rho_{41}(0) & 0 & 0 & \rho_{44}(0) \end{bmatrix}$$

. . . it is preserved during the evolution

$$\rho(t) = \begin{bmatrix} \rho_{11}(t) & 0 & 0 & \rho_{14}(t) \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0 \\ \rho_{41}(t) & 0 & 0 & \rho_{44}(t) \end{bmatrix}$$

1.4 Collective states basis

In the collective basis the density matrix has also the X form ...

$$\rho(0) = \begin{bmatrix} \rho_{gg}(0) & 0 & 0 & \rho_{ge}(0) \\ 0 & \rho_{ss}(0) & \rho_{sa}(0) & 0 \\ 0 & \rho_{as}(0) & \rho_{aa}(0) & 0 \\ \rho_{eg}(0) & 0 & 0 & \rho_{ee}(0) \end{bmatrix}$$

1.4 Collective states basis

In the collective basis the density matrix has also the X form ...

$$\rho(0) = \begin{bmatrix} \rho_{gg}(0) & 0 & 0 & \rho_{ge}(0) \\ 0 & \rho_{ss}(0) & \rho_{sa}(0) & 0 \\ 0 & \rho_{as}(0) & \rho_{aa}(0) & 0 \\ \rho_{eg}(0) & 0 & 0 & \rho_{ee}(0) \end{bmatrix}$$

... which is preserved during the evolution

$$\rho(t) = \begin{bmatrix} \rho_{gg}(t) & 0 & 0 & \rho_{ge}(t) \\ 0 & \rho_{ss}(t) & \rho_{sa}(t) & 0 \\ 0 & \rho_{as}(t) & \rho_{aa}(t) & 0 \\ \rho_{eg}(t) & 0 & 0 & \rho_{ee}(t) \end{bmatrix}$$

1.5 Analytical solutions

$$\rho_{ss}(t) = \rho_{ss}(0)\,\mathrm{e}^{-(\Gamma+\Gamma_{12})t} + \rho_{ee}(0)\frac{\Gamma+\Gamma_{12}}{\Gamma-\Gamma_{12}}\left(\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right)$$

$$\rho_{aa}(t) = \rho_{aa}(0)\,\mathrm{e}^{-(\Gamma-\Gamma_{12})t} + \rho_{ee}(0)\,\frac{\Gamma-\Gamma_{12}}{\Gamma+\Gamma_{12}}\left(\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right)$$

$$\rho_{as}(t) = \rho_{as}(0)\,\mathrm{e}^{-(\Gamma+i2\Omega_{12})t}$$

$$\rho_{ee}(t) = \rho_{ee}(0)\,\mathrm{e}^{-2\Gamma t}$$

$$\rho_{eg}(t) = \rho_{eg}(0)\,\mathrm{e}^{-(\Gamma+2i\omega_0)t}$$

$$\rho_{gg}(t) = 1 - \rho_{ee}(t) - \rho_{ss}(t) - \rho_{aa}(t)$$

1.5 Analytical solutions

$$\rho_{ss}(t) = \rho_{ss}(0)\,\mathrm{e}^{-(\Gamma+\Gamma_{12})t} + \rho_{ee}(0)\frac{\Gamma+\Gamma_{12}}{\Gamma-\Gamma_{12}}\left(\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right)$$

$$\rho_{aa}(t) = \rho_{aa}(0)\,\mathrm{e}^{-(\Gamma-\Gamma_{12})t} + \rho_{ee}(0)\,\frac{\Gamma-\Gamma_{12}}{\Gamma+\Gamma_{12}}\left(\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right)$$

$$\rho_{as}(t) = \rho_{as}(0)\,\mathrm{e}^{-(\Gamma+i2\Omega_{12})t}$$

$$\rho_{ee}(t) = \rho_{ee}(0)\,\mathrm{e}^{-2\Gamma t}$$

$$\rho_{eg}(t) = \rho_{eg}(0)\,\mathrm{e}^{-(\Gamma+2i\omega_0)t}$$

$$\rho_{gg}(t) = 1 - \rho_{ee}(t) - \rho_{ss}(t) - \rho_{aa}(t)$$

$$\boxed{\rho_{ij}(t\to\infty)\to 0 \quad (\rho_{ij}\neq\rho_{gg})}$$

2 Concurrence

W.K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998)

2 Concurrence

W.K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998)

$$\mathcal{C} = \max\left(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right)$$

$\{\lambda_i\}$ — eigenvalues on the matrix R

$$R = \rho\left(\sigma_y \otimes \sigma_y \, \rho^* \, \sigma_y \otimes \sigma_y\right)$$

2 Concurrence

W.K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998)

$$\mathcal{C} = \max\left(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right)$$

$\{\lambda_i\}$ — eigenvalues on the matrix R

$$R = \rho\left(\sigma_y \otimes \sigma_y\, \rho^*\, \sigma_y \otimes \sigma_y\right)$$

$$\boxed{0 \leq \mathcal{C} \leq 1}$$

$\mathcal{C} = 0$ no entanglement

$\mathcal{C} = 1$ maximal entanglement

Analytical solutions for concurrence:

$$\mathcal{C}(t) = \max\left\{0, \mathsf{C}_1(t), \mathsf{C}_2(t)\right\}$$

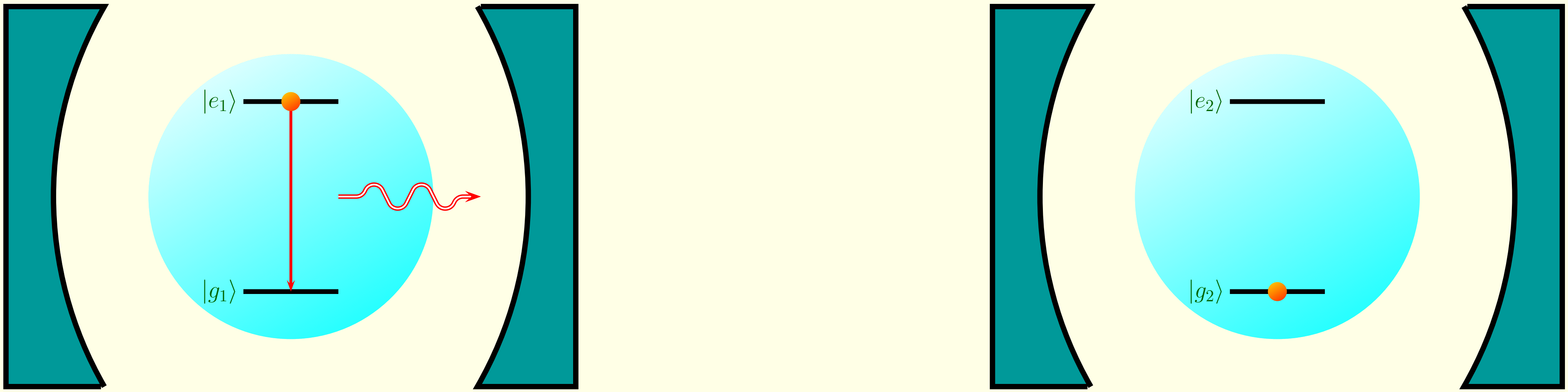
$$\mathsf{C}_1(t) = 2|\rho_{ge}(t)| - \sqrt{\left[\rho_{ss}(t) + \rho_{aa}(t)\right]^2 - \left[2\Re\rho_{sa}(t)\right]^2}$$

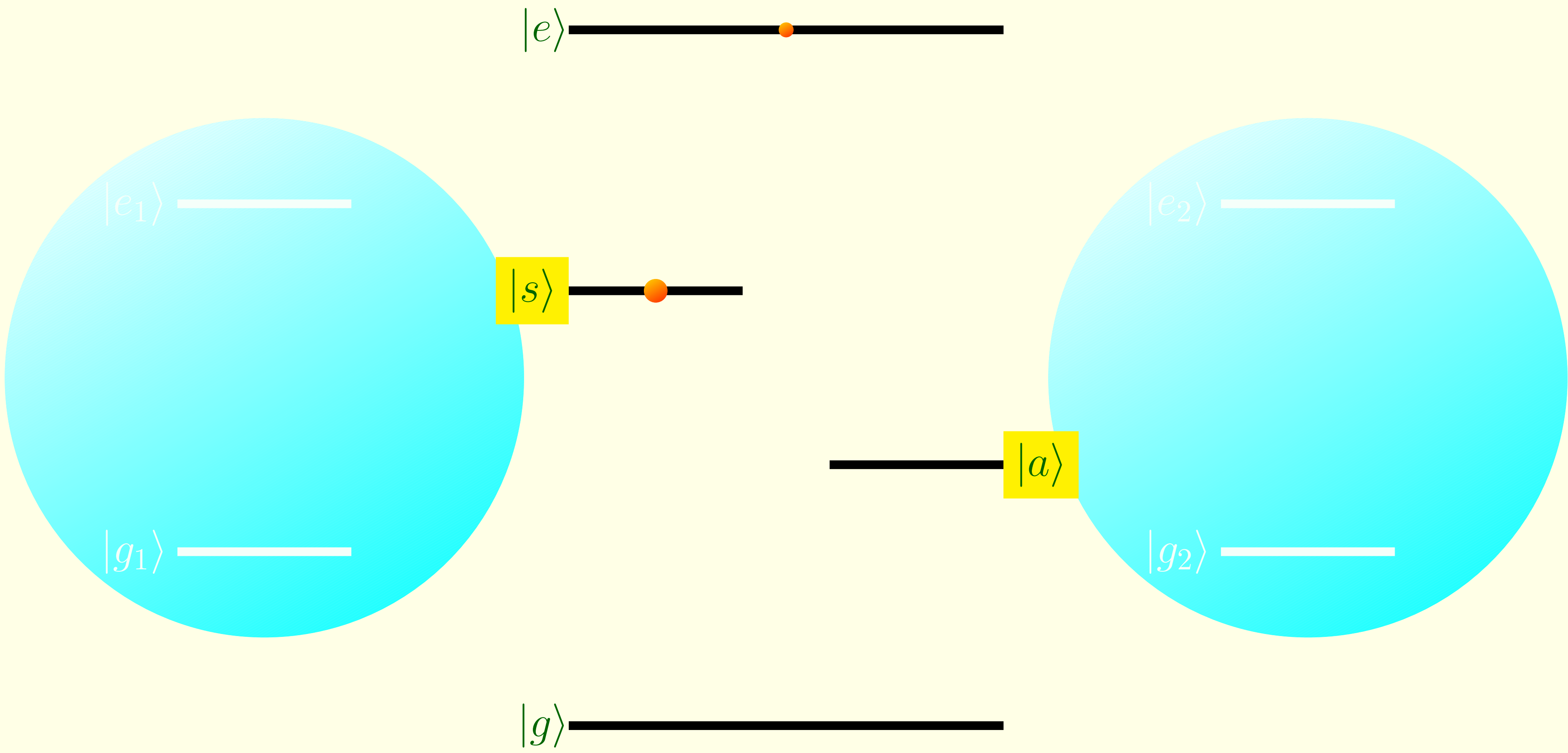
$$\mathsf{C}_2(t) = \sqrt{\left[\rho_{ss}(t) - \rho_{aa}(t)\right]^2 + \left[2\Im\rho_{sa}(t)\right]^2} - 2\sqrt{\rho_{ee}(t)\rho_{gg}(t)}$$

3 Dynamics of entanglement

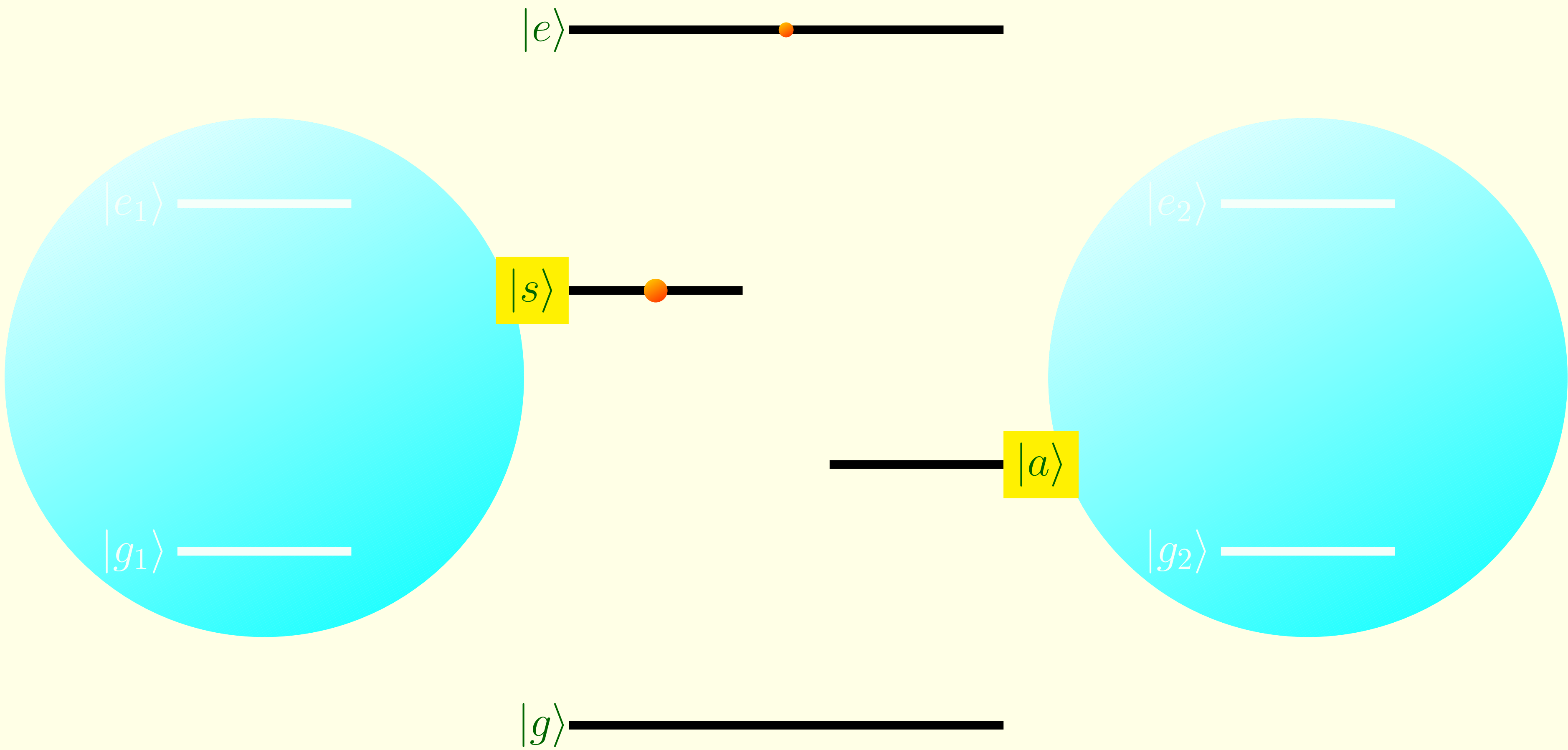
3.1 Entanglement sudden death and revival

T. Yu, J. H. Eberly, Phys. Rev. Lett. **93**, 140404 (2004)



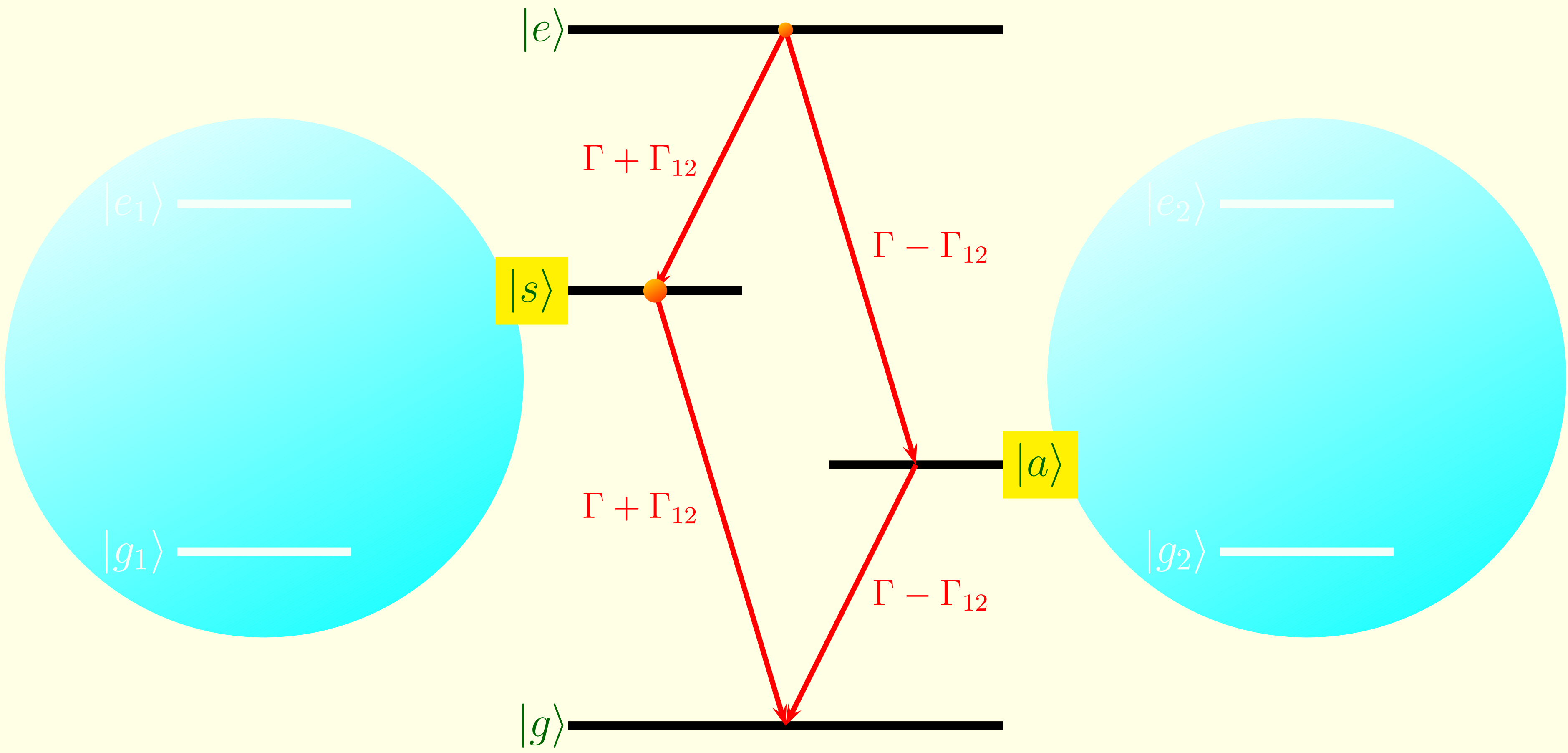


$\rho_{ss}(0) = \frac{2}{3}$, $\rho_{gg}(0) = \frac{1}{3}(1-\alpha)$, $\rho_{ee}(0) = \frac{1}{3}\alpha$



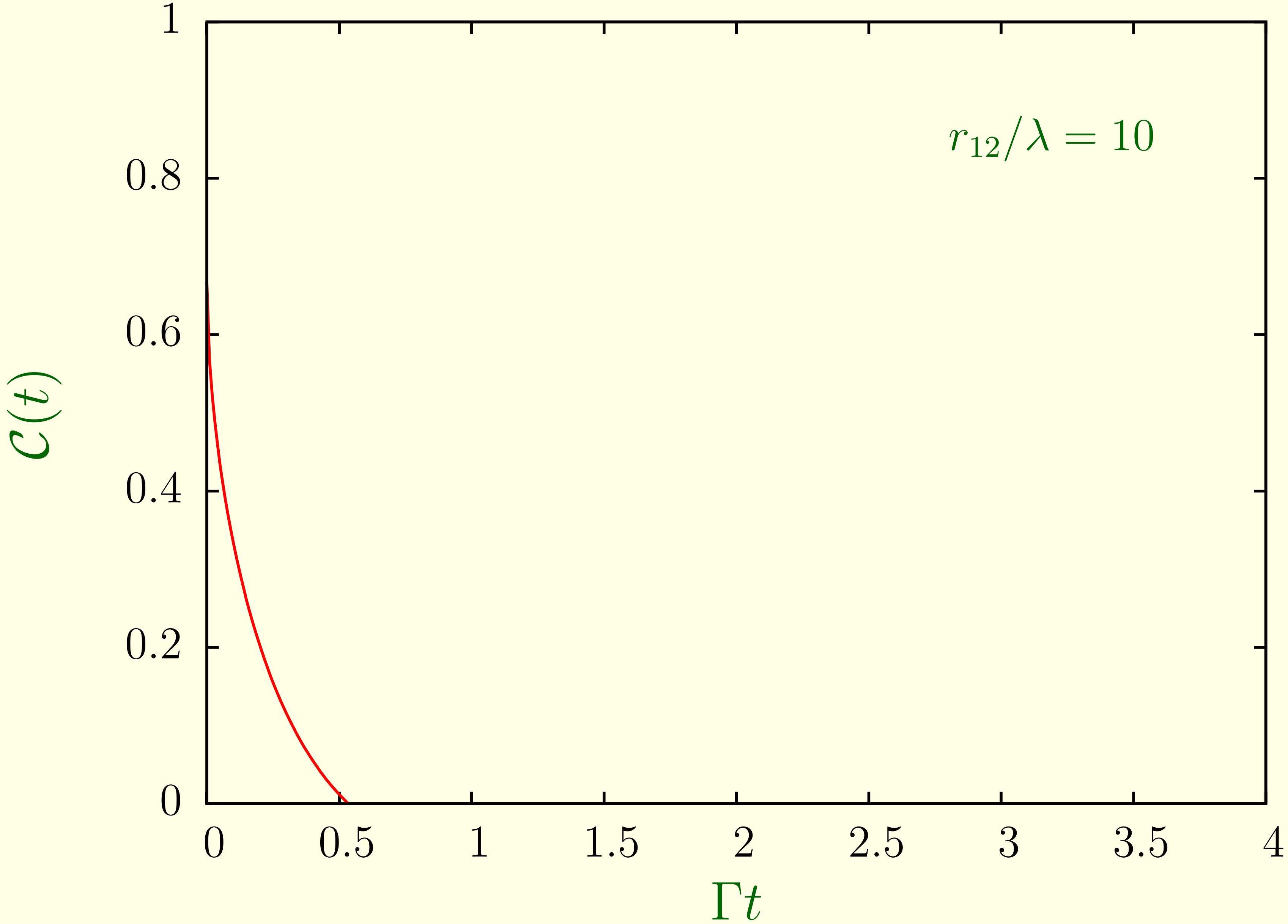
$$\rho_{ss}(0) = \frac{2}{3}, \quad \rho_{gg}(0) = \frac{1}{3}(1-\alpha), \quad \rho_{ee}(0) = \frac{1}{3}\alpha$$

$$\boxed{\mathcal{C}(0) = \frac{2}{3}\left(1 - \sqrt{\alpha(1-\alpha)}\right)}$$

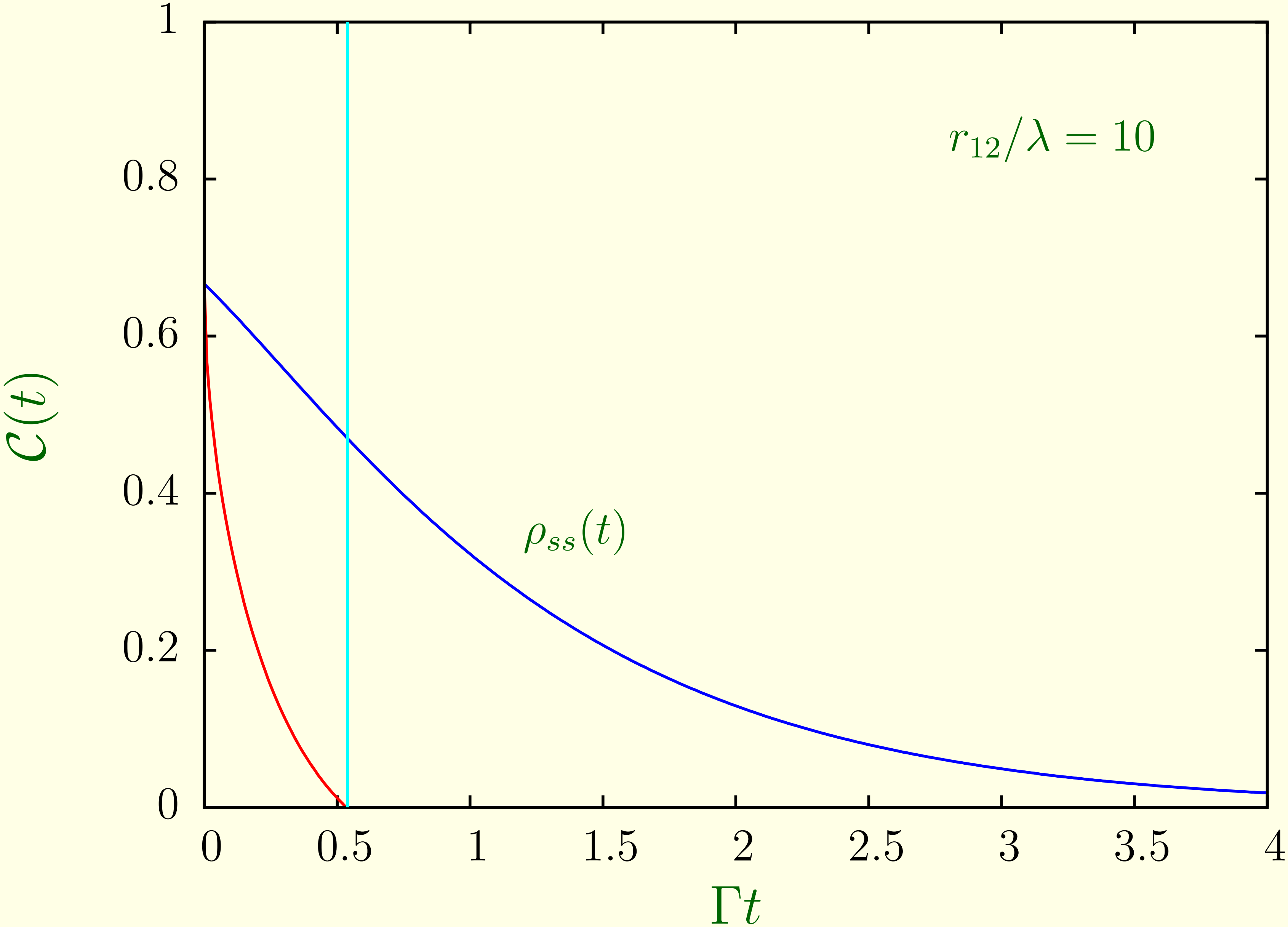


$$\mathsf{C}_2(t) = |\rho_{ss}(t) - \rho_{aa}(t)| - 2\sqrt{\rho_{gg}(t)\,\rho_{ee}(t)}$$

Sudden death



Sudden death



For $\alpha = 1$ entanglement disapears after time

$$t_d = \frac{1}{\Gamma} \ln \left(\frac{2 + \sqrt{2}}{2} \right) ,$$

which is finite despite the fact that all matrix elements decay only asymptotically when $t \to \infty$.

Time t_d is finite for $\frac{1}{3} < \alpha \leq 1$, and for $\alpha < \frac{1}{3}$ entanglement decays asymptotically.

For $\alpha = 1$ entanglement disapears after time

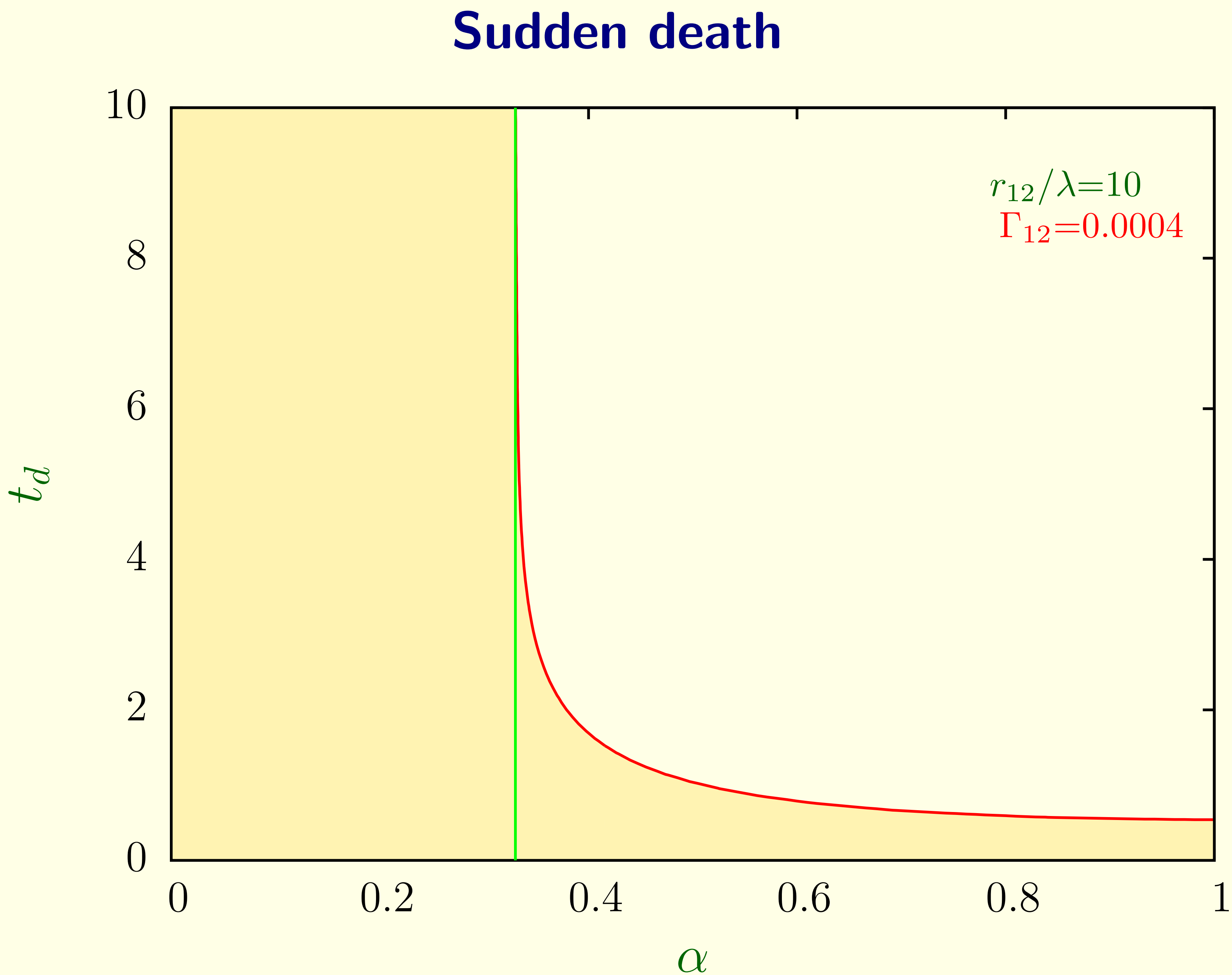
$$t_d = \frac{1}{\Gamma} \ln \left(\frac{2 + \sqrt{2}}{2} \right) ,$$

which is finite despite the fact that all matrix elements decay only asymptotically when $t \to \infty$.

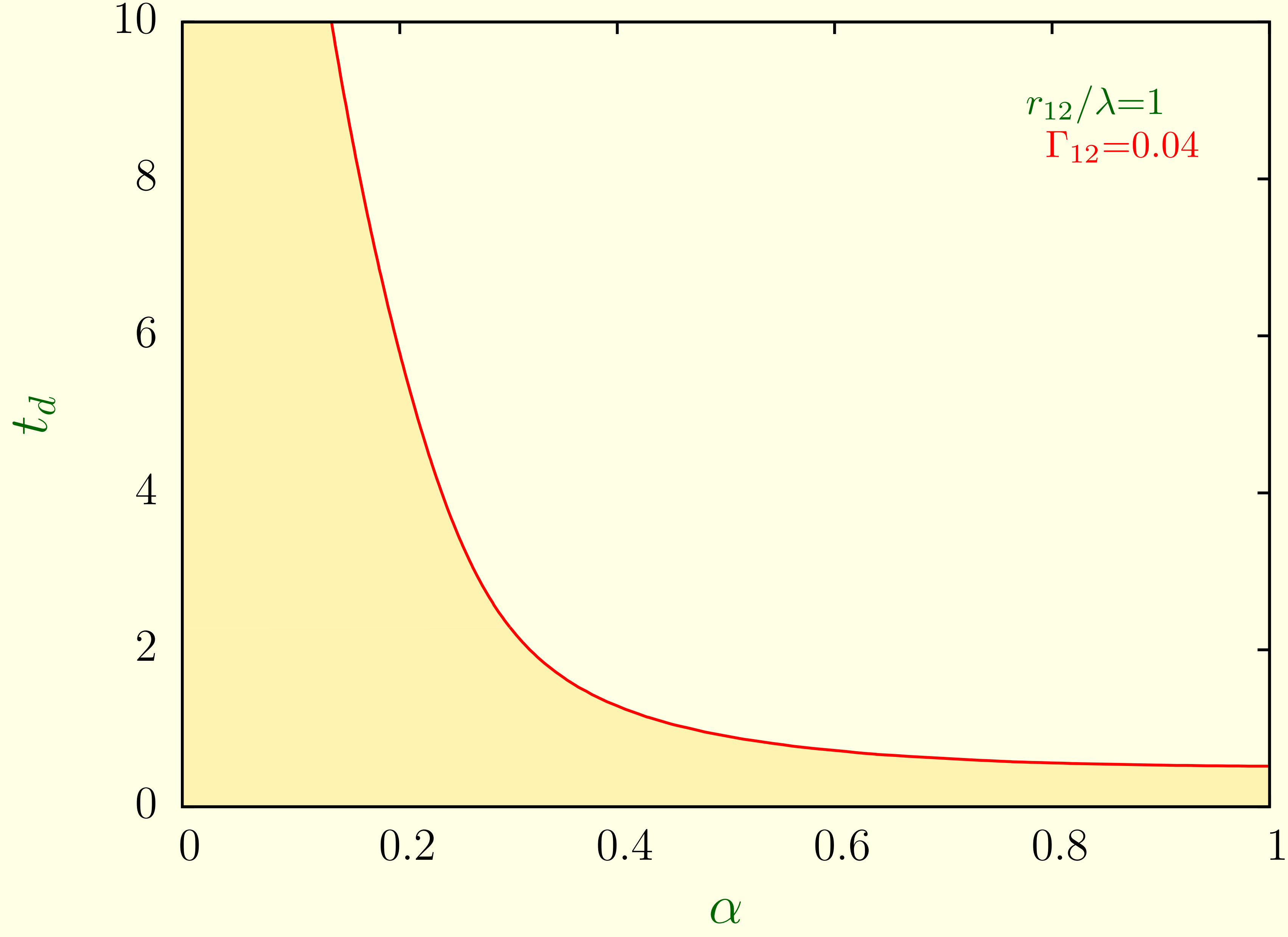
Time t_d is finite for $\frac{1}{3} < \alpha \leq 1$, and for $\alpha < \frac{1}{3}$ entanglement decays asymptotically.

How the sudden death looks like for interacting atoms?

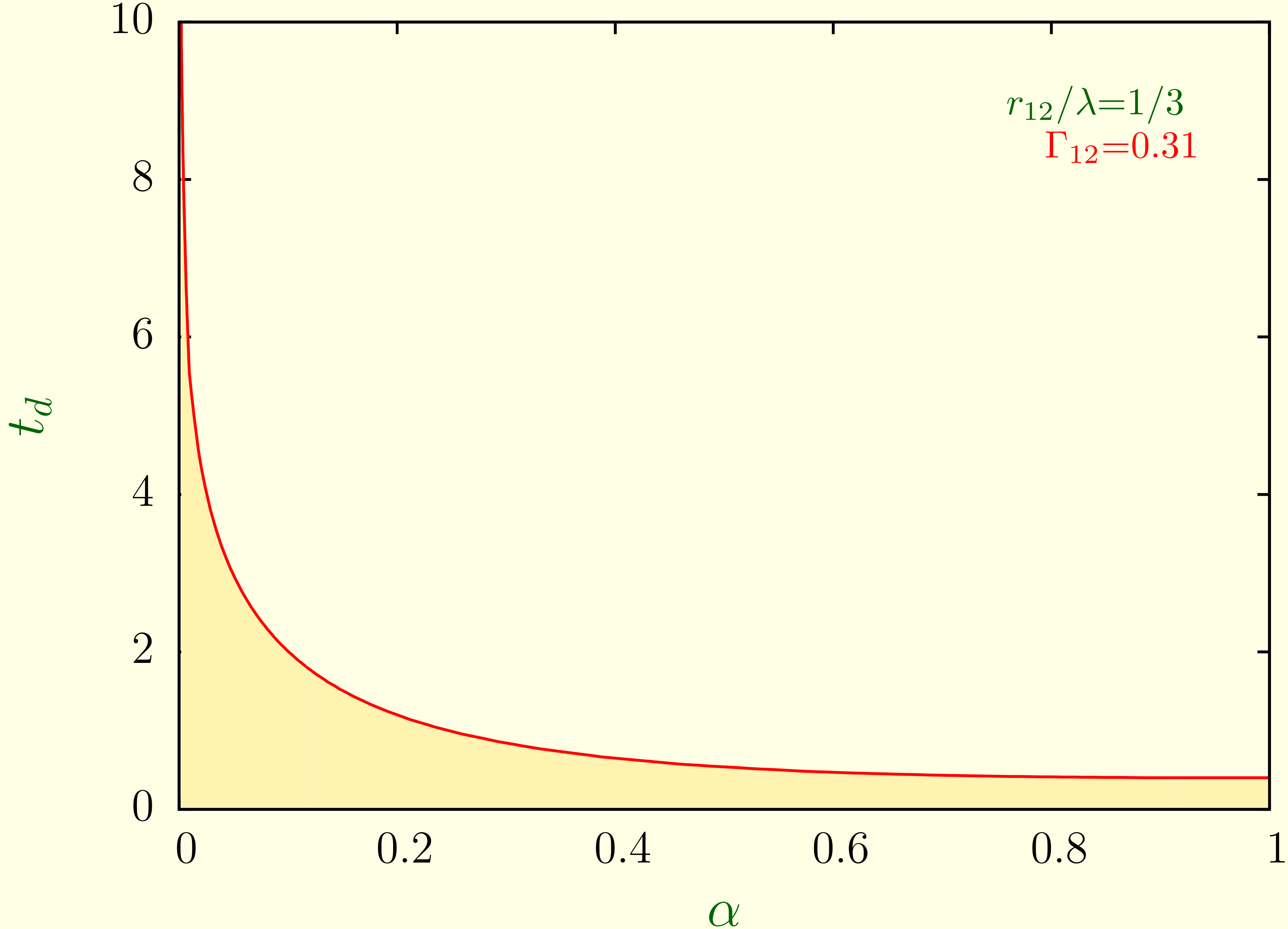
3.2 Sudden death and revival of entanglement

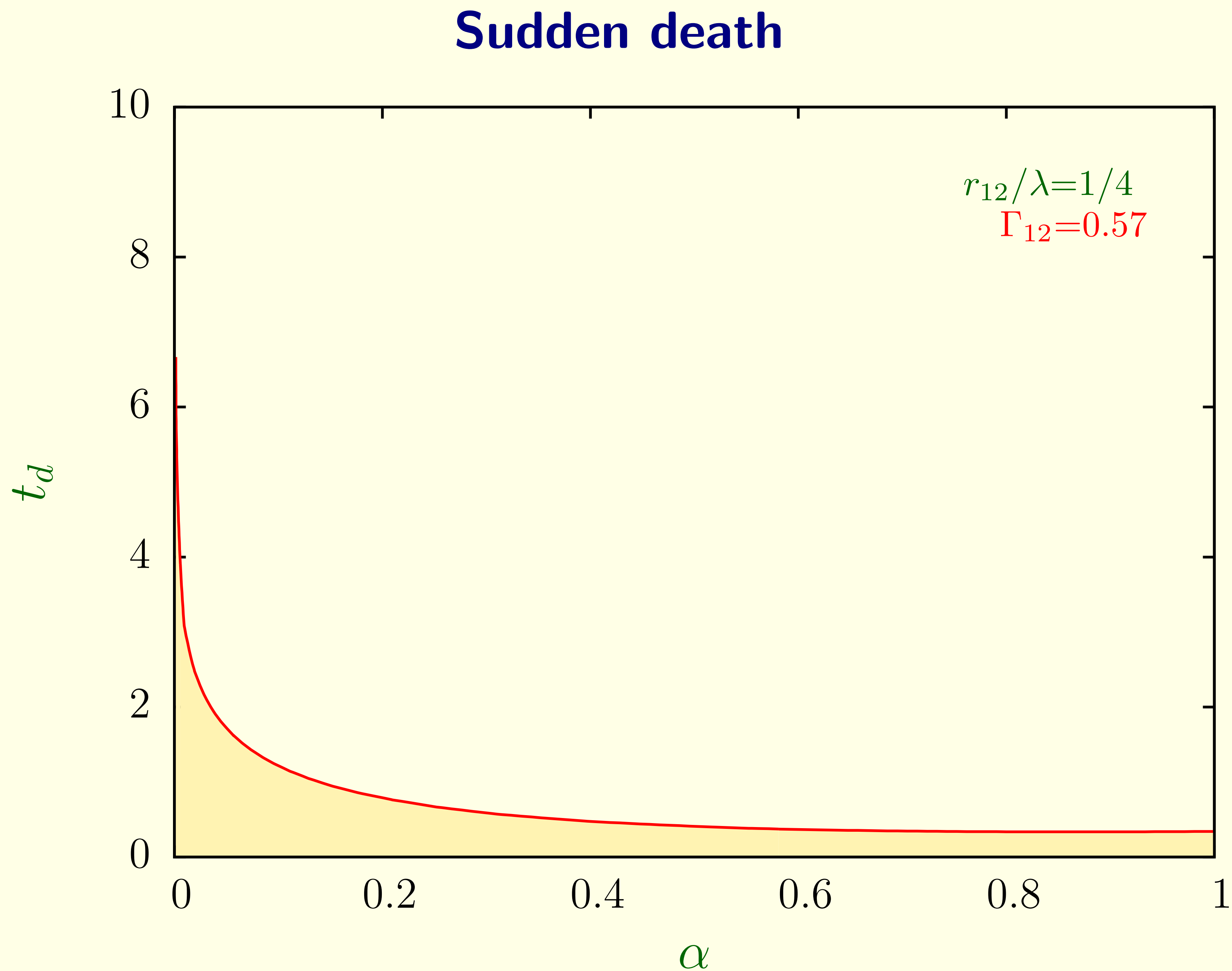


Sudden death

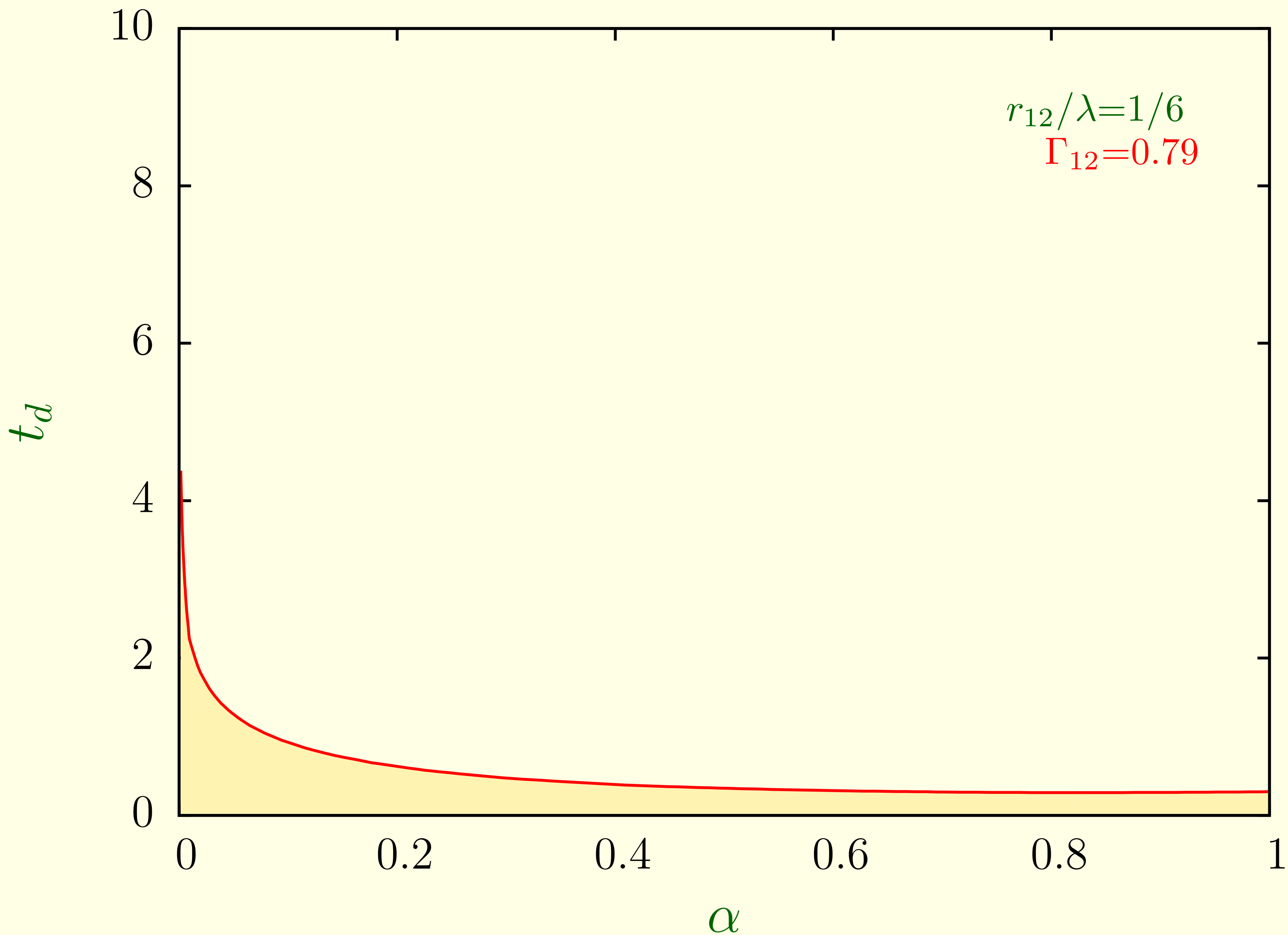


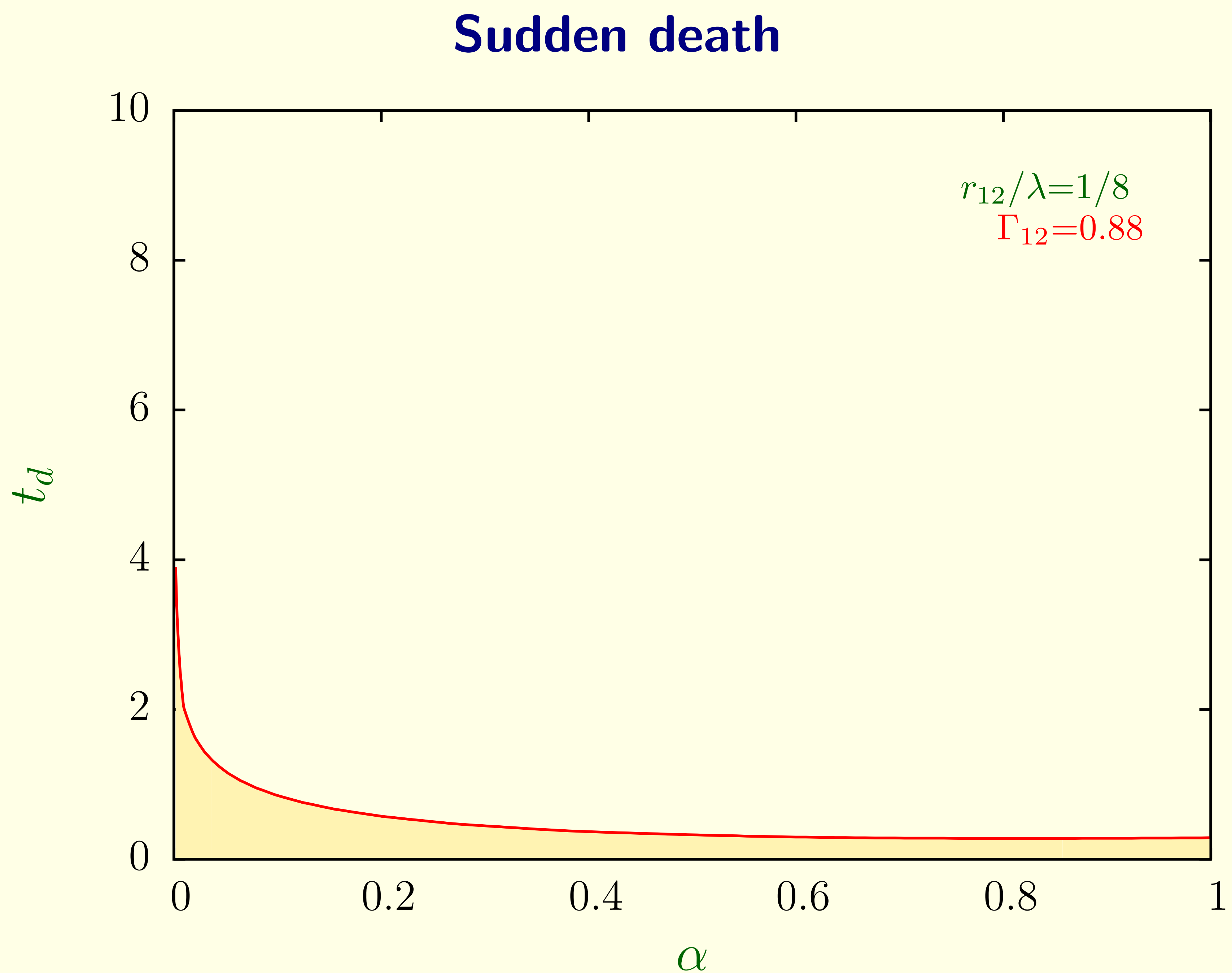
Sudden death




Sudden death
r_{12}/λ=1/4
Γ_{12}=0.57
t_d
α
0
2
4
6
8
10
0
0.2
0.4
0.6
0.8
1

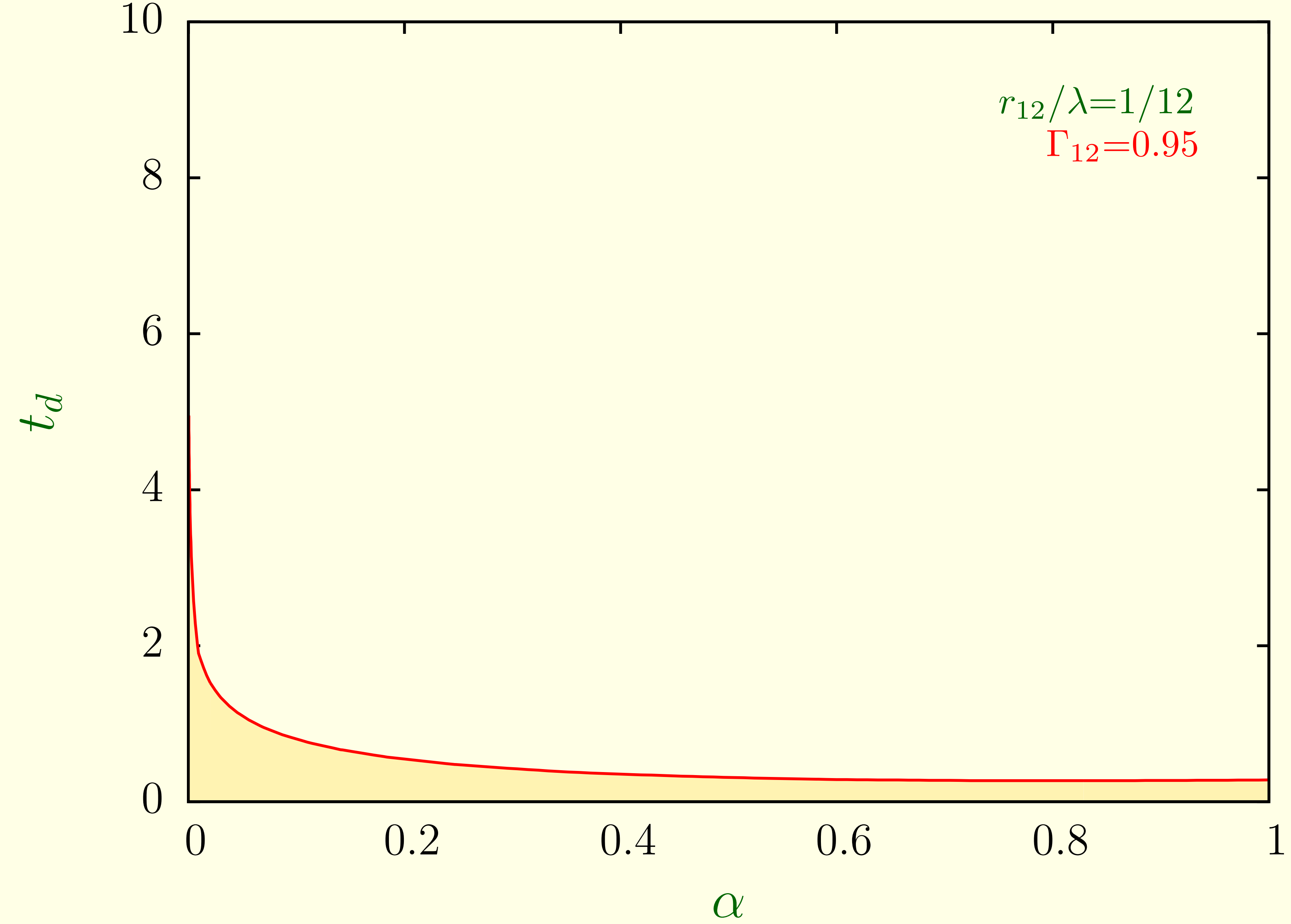
Sudden death





Sudden death
r_{12}/λ=1/8
Γ_{12}=0.88
t_d
α
0
2
4
6
8
10
0
0.2
0.4
0.6
0.8
1

Sudden death



Can entanglement revive?

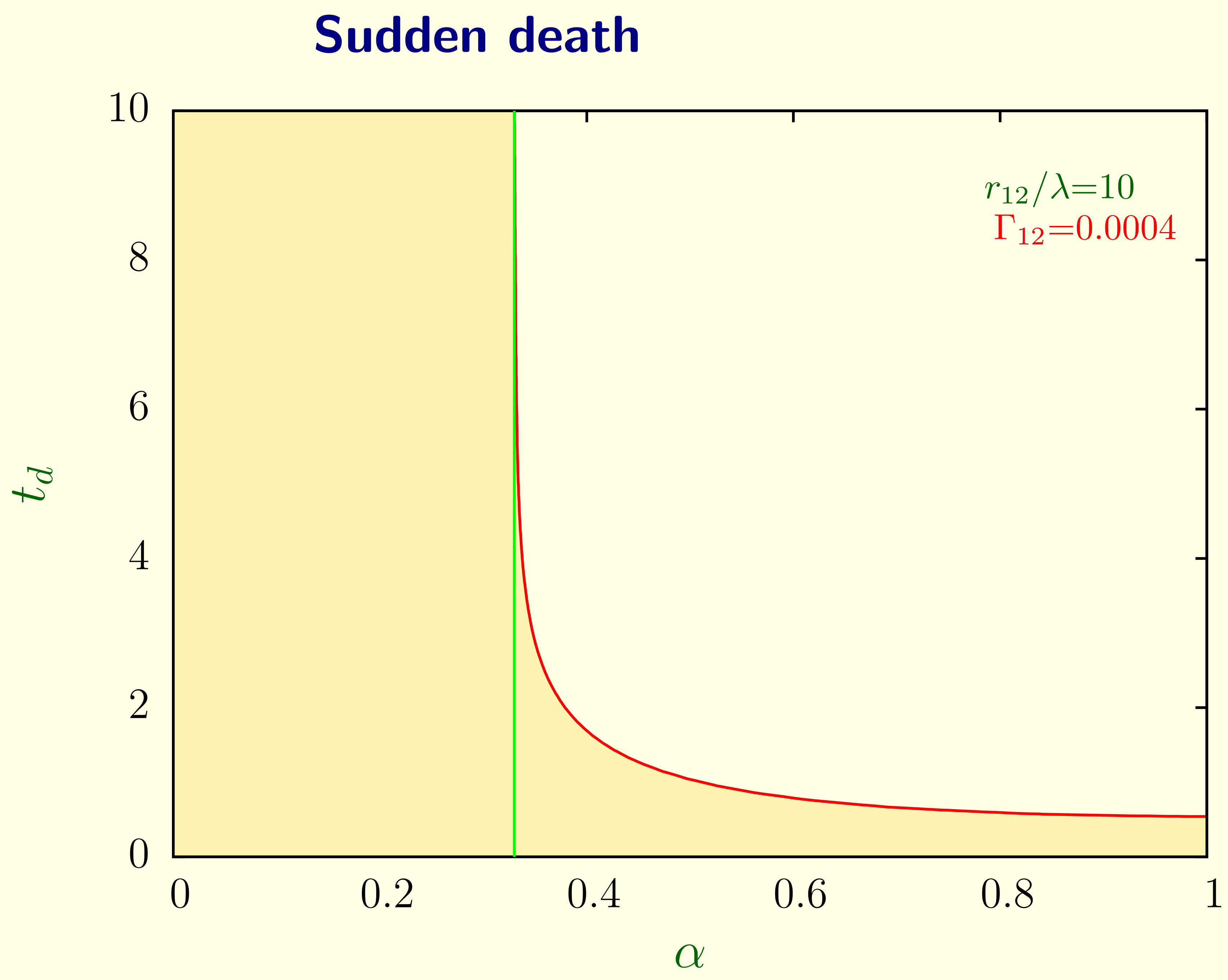
$$\rho_{gg}(t) = 1 - [\rho_{ee}(t) + \rho_{ss}(t) + \rho_{aa}(t)]$$

$$\rho_{ee}(t) = \frac{\alpha}{3}\, e^{-2\Gamma t}$$

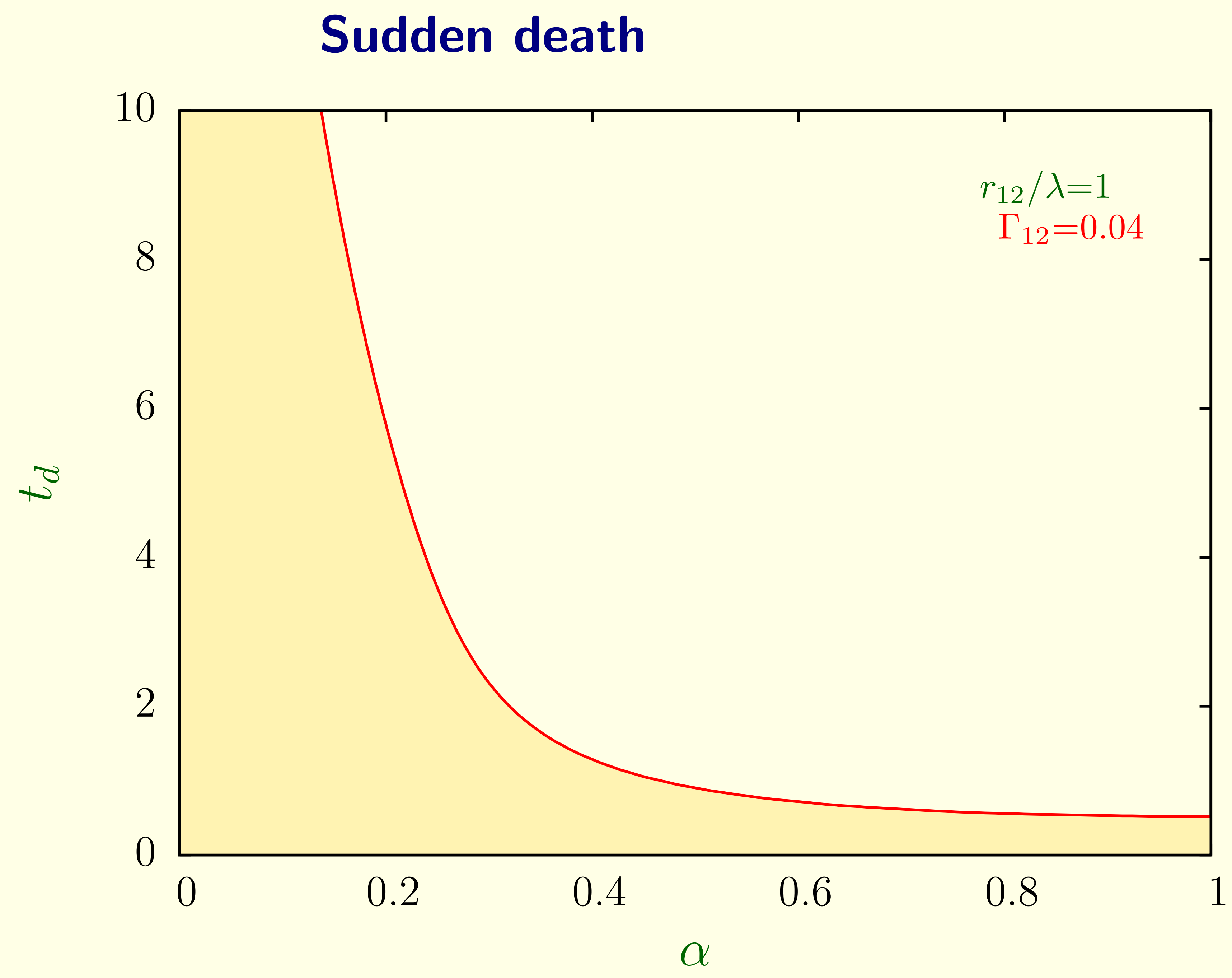
$$\rho_{ss}(t) = \frac{2}{3} e^{-(\Gamma+\Gamma_{12})t} + \frac{\alpha}{3}\frac{\Gamma+\Gamma_{12}}{\Gamma-\Gamma_{12}}\left[e^{-(\Gamma+\Gamma_{12})t} - e^{-2\Gamma t}\right]$$

$$\rho_{aa}(t) = \frac{\alpha}{3}\frac{\Gamma-\Gamma_{12}}{\Gamma+\Gamma_{12}}\left[e^{-(\Gamma-\Gamma_{12})t} - e^{-2\Gamma t}\right]$$

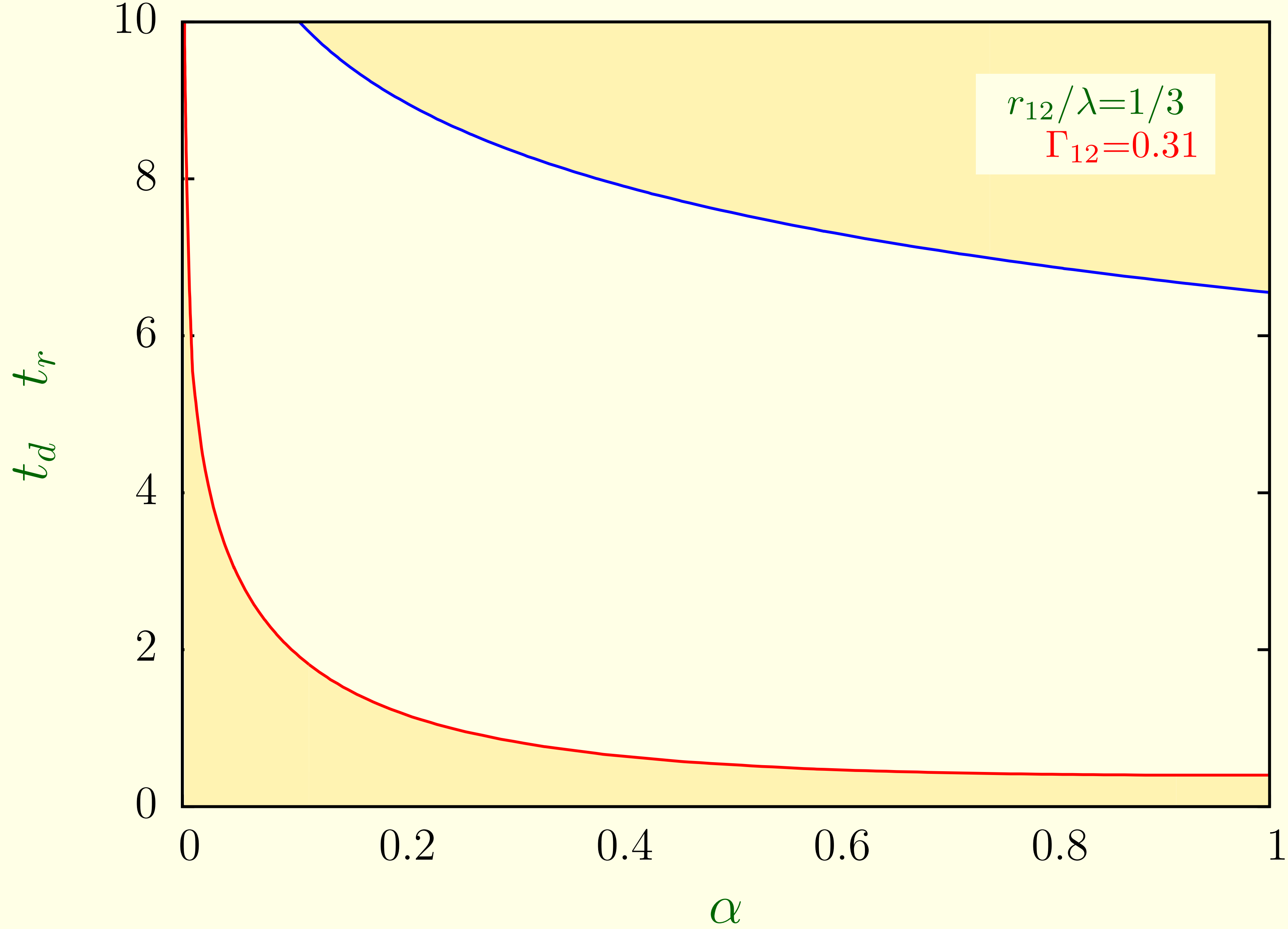
$$\mathsf{C}_2(t) = |\rho_{ss}(t) - \rho_{aa}(t)| - 2\sqrt{\rho_{gg}(t)\rho_{ee}(t)}$$



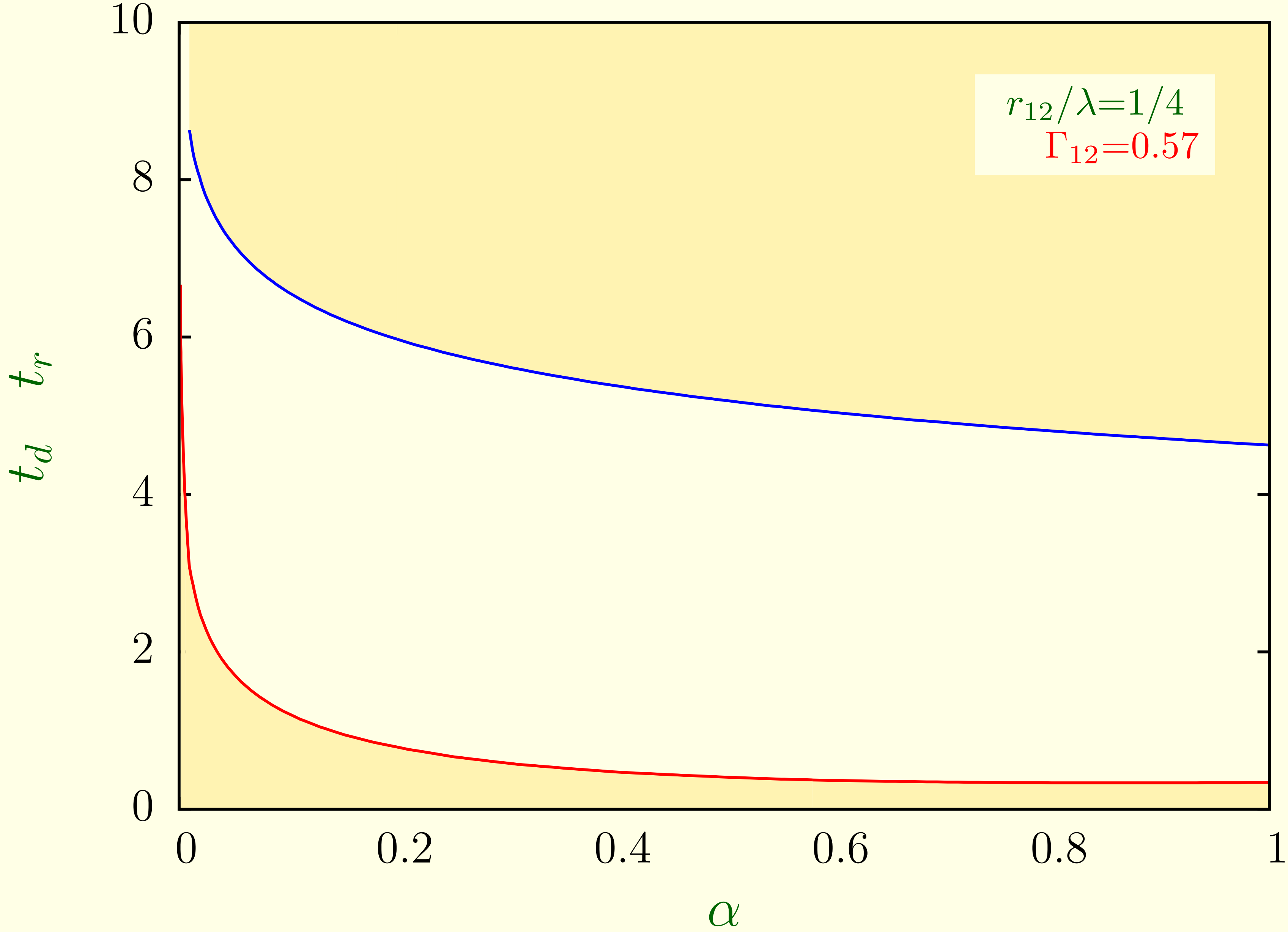
Sudden death
r_{12}/λ=10
Γ_{12}=0.0004
t_d
α
0
2
4
6
8
10
0
0.2
0.4
0.6
0.8
1


Sudden death
r_{12}/λ=1
Γ_{12}=0.04
t_d
α
0
2
4
6
8
10
0
0.2
0.4
0.6
0.8
1

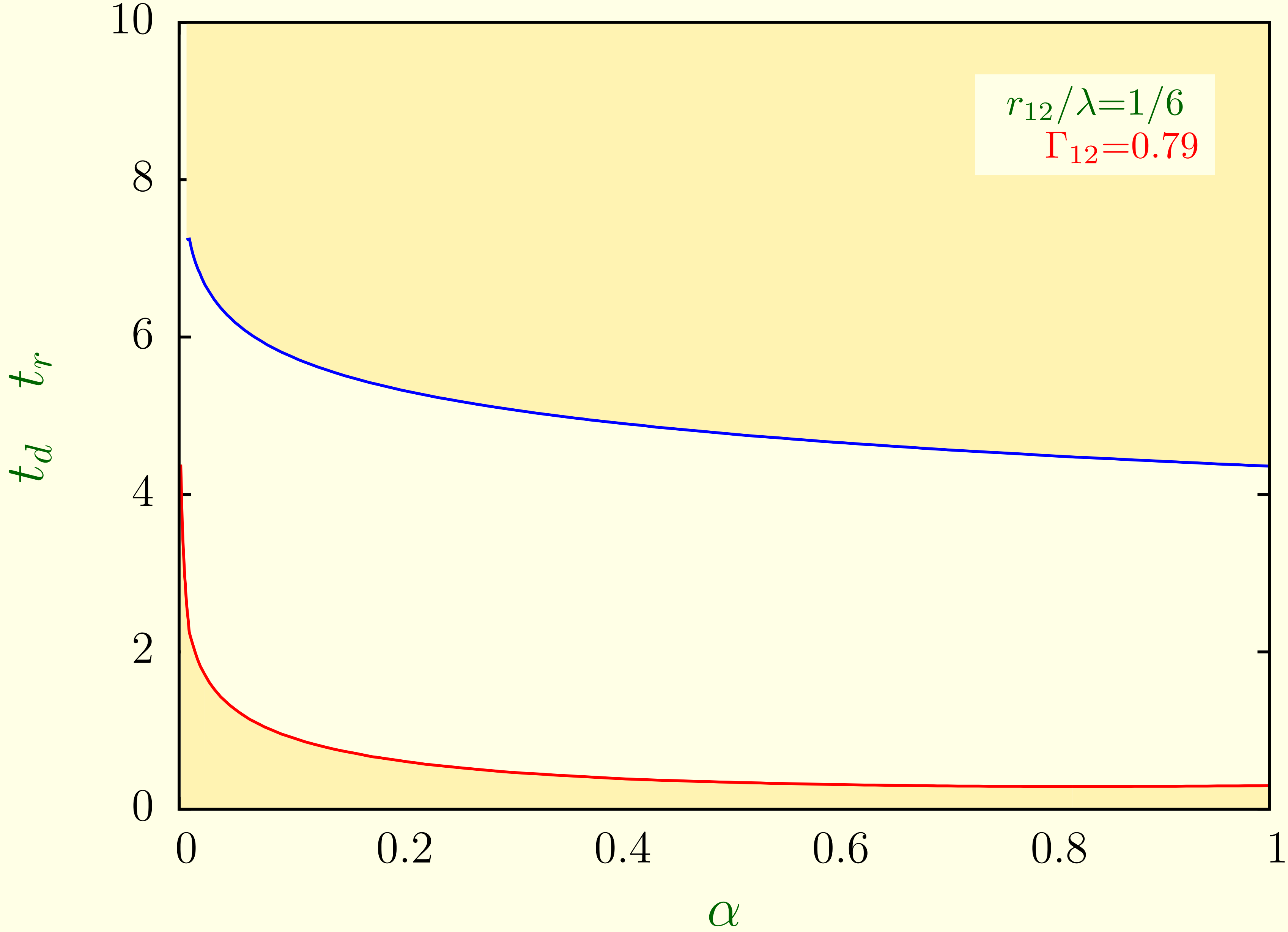
Sudden death and revival



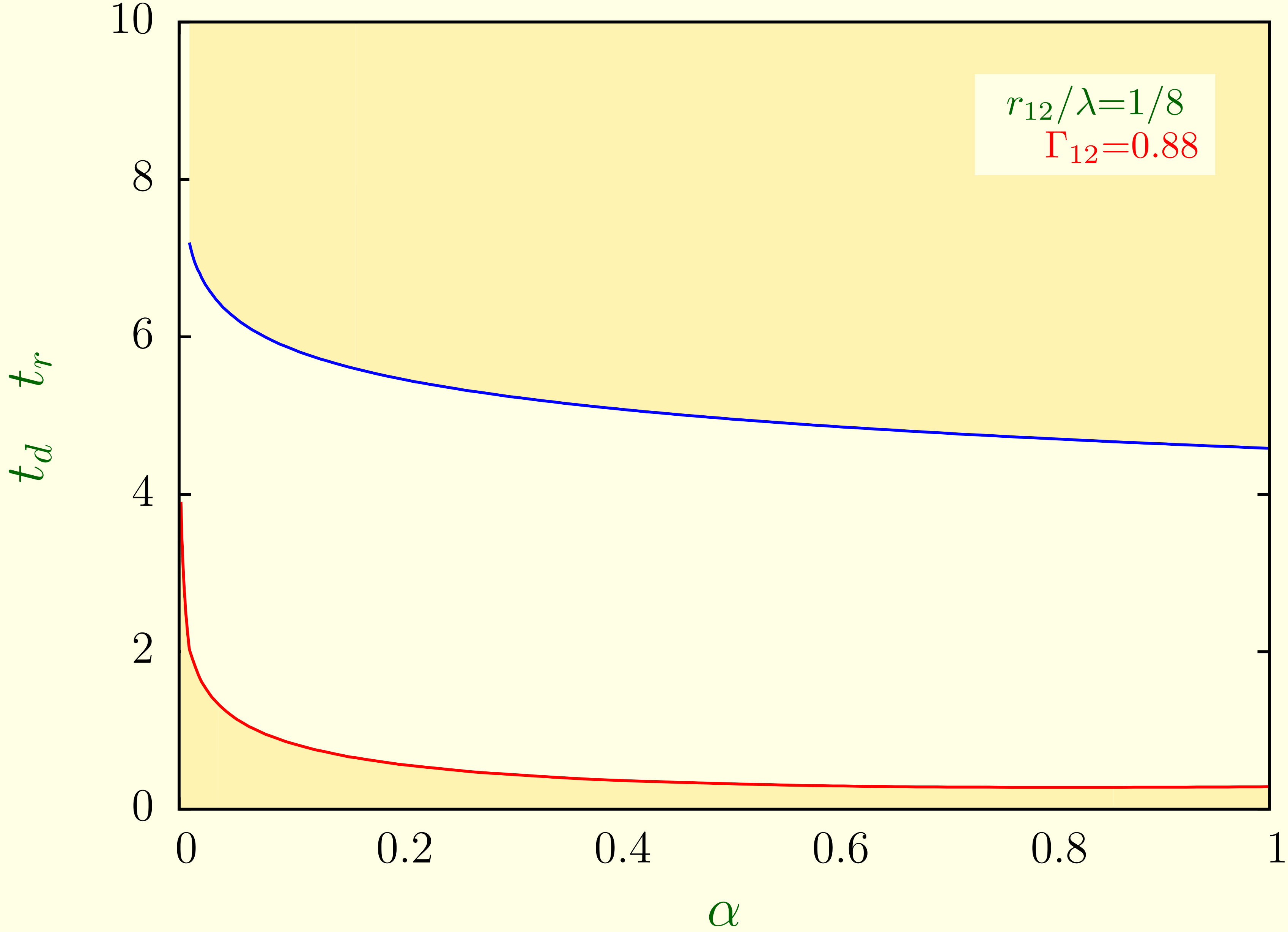
Sudden death and revival

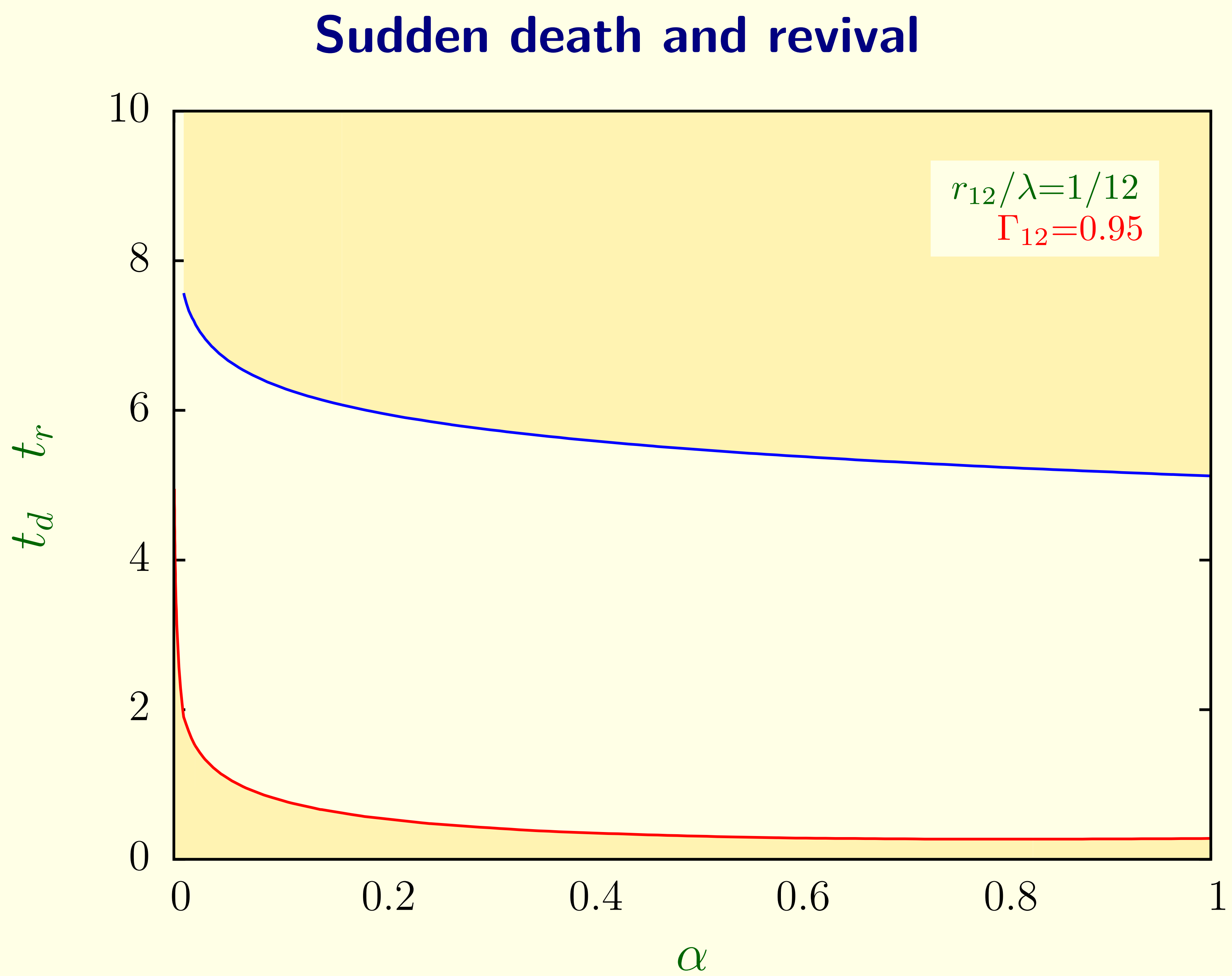


Sudden death and revival



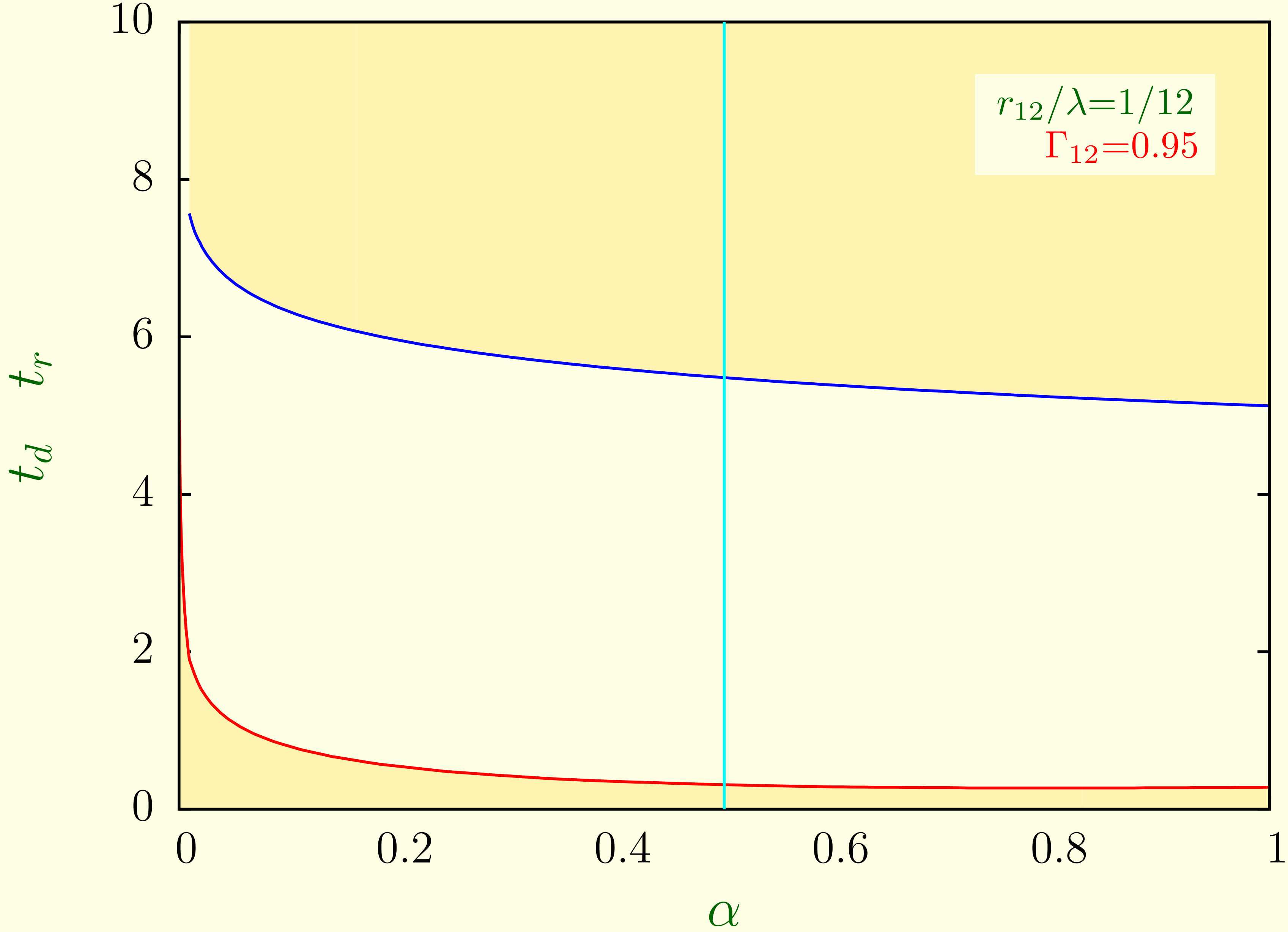
Sudden death and revival



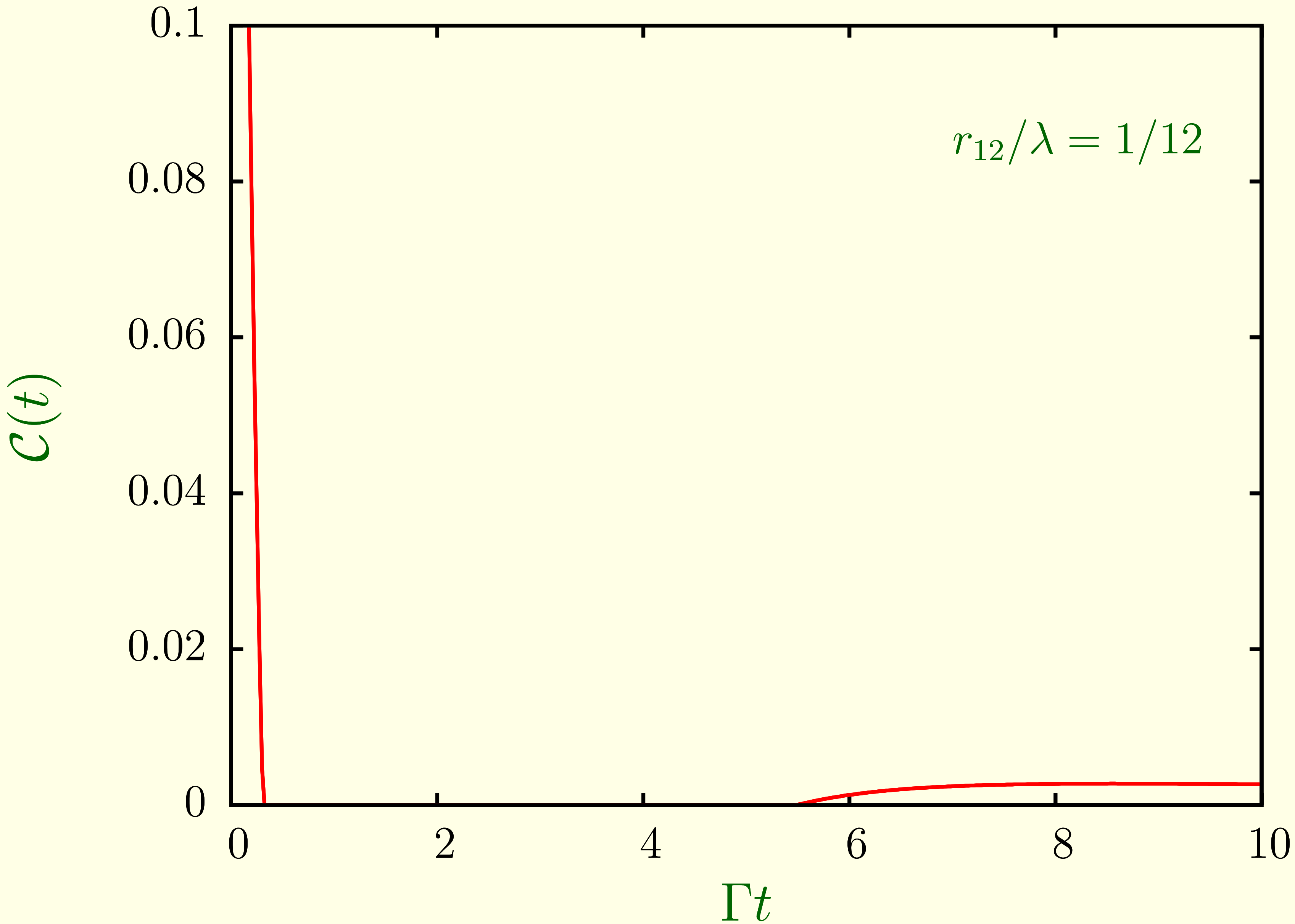


Sudden death and revival
10
8
6
4
2
0
0
0.2
0.4
0.6
0.8
1
$t_d \quad t_r$
α
r_{12}/λ=1/12
Γ_{12}=0.95

Sudden death and revival



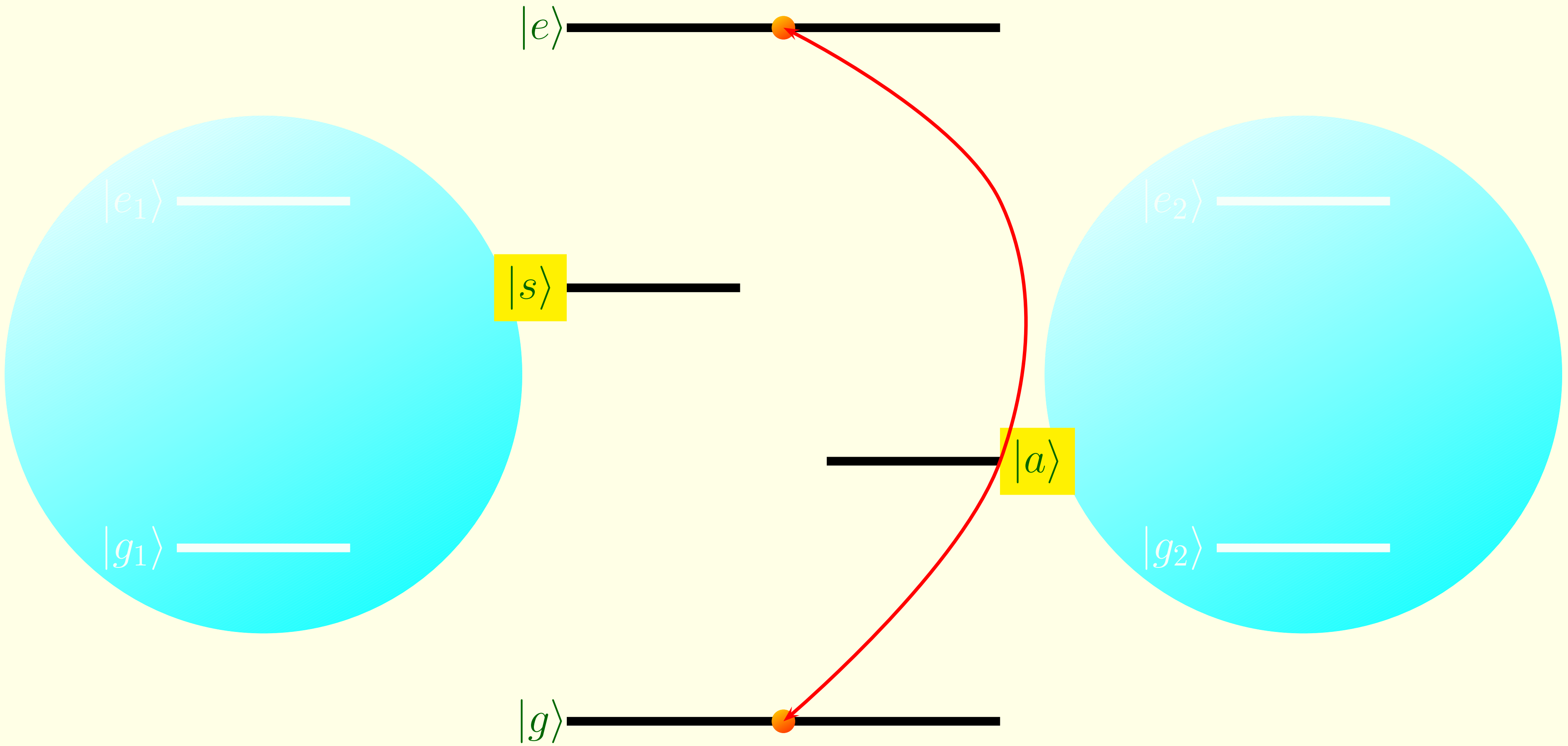
Sudden death and revival



Another example

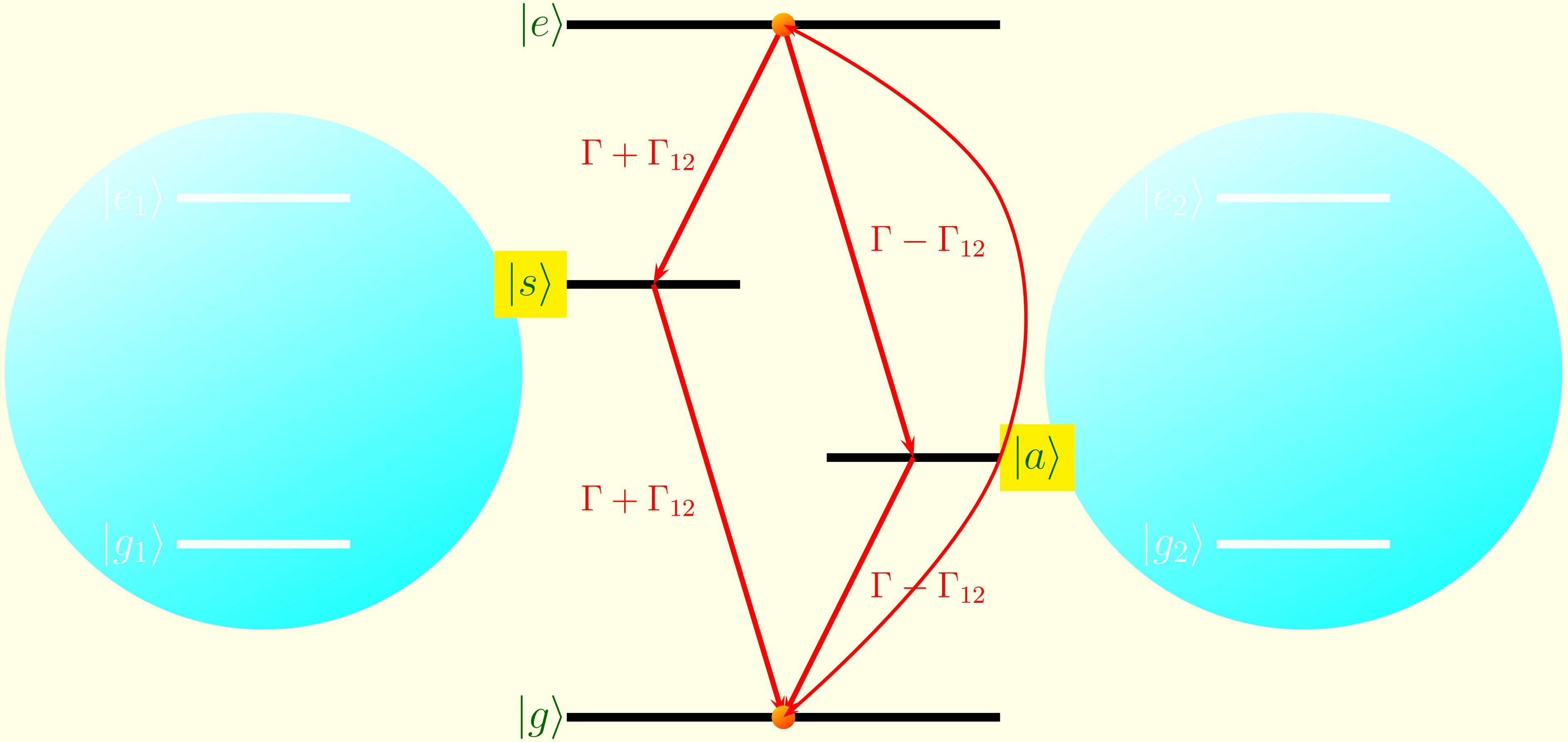
Z. Ficek, R. Tanaś, Phys. Rev. A **74**, 024304 (2006)

$$|\Psi_0\rangle = \sqrt{p}\,|e\rangle + \sqrt{1-p}\,|g\rangle, \quad \mathcal{C}(0) = 2\sqrt{p(1-p)}$$



Z. Ficek, R. Tanaś, Phys. Rev. A, **74**, 024304 (2006)

$$|\Psi_0\rangle = \sqrt{p}\,|e\rangle + \sqrt{1-p}\,|g\rangle, \quad \mathcal{C}(0) = 2\sqrt{p(1-p)}$$



$$\mathsf{C}_1(t) = 2\,|\rho_{ge}(t)| - [\rho_{ss}(t) + \rho_{aa}(t)]$$

$$\mathsf{C}_2(t) = |\rho_{ss}(t) - \rho_{aa}(t)| - 2\sqrt{\rho_{gg}(t)\rho_{ee}(t)}$$

$$\mathsf{C}_1(t) = 2\,|\rho_{ge}(t)| - [\rho_{ss}(t) + \rho_{aa}(t)]$$

$$\mathsf{C}_2(t) = |\rho_{ss}(t) - \rho_{aa}(t)| - 2\sqrt{\rho_{gg}(t)\rho_{ee}(t)}$$

$$\rho_{ee}(t) = p\,\mathrm{e}^{-2\Gamma t}$$

$$|\rho_{ge}(t)| = \sqrt{p(1-p)}\,\mathrm{e}^{-\Gamma t}$$

$$\rho_{ss}(t) = p\,\frac{\Gamma+\Gamma_{12}}{\Gamma-\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\rho_{aa}(t) = p\,\frac{\Gamma-\Gamma_{12}}{\Gamma+\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

Independent atoms: $\Gamma_{12} = 0$

$$\rho_{ee}(t) = p\,\mathrm{e}^{-2\Gamma t}$$

$$|\rho_{ge}(t)| = \sqrt{p(1-p)}\,\mathrm{e}^{-\Gamma t}$$

$$\rho_{ss}(t) = \rho_{aa}(t) = p\,\left[\mathrm{e}^{-\Gamma t} - \mathrm{e}^{-2\Gamma t}\right]$$

Independent atoms: $\Gamma_{12} = 0$

$$\rho_{ee}(t) = p\,\mathrm{e}^{-2\Gamma t}$$

$$|\rho_{ge}(t)| = \sqrt{p(1-p)}\,\mathrm{e}^{-\Gamma t}$$

$$\rho_{ss}(t) = \rho_{aa}(t) = p\,\left[\mathrm{e}^{-\Gamma t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\begin{aligned} \mathsf{C}_1(t) &= 2\sqrt{p(1-p)}\,\mathrm{e}^{-\Gamma t} - 2p\,\left[\mathrm{e}^{-\Gamma t} - \mathrm{e}^{-2\Gamma t}\right] \\ \mathsf{C}_2(t) &= -2\sqrt{\rho_{gg}(t)\rho_{ee}(t)} < 0 \end{aligned}$$

Independent atoms: $\Gamma_{12} = 0$

$$\rho_{ee}(t) = p\,\mathrm{e}^{-2\Gamma t}$$

$$|\rho_{ge}(t)| = \sqrt{p(1-p)}\,\mathrm{e}^{-\Gamma t}$$

$$\rho_{ss}(t) = \rho_{aa}(t) = p\,\left[\mathrm{e}^{-\Gamma t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\mathsf{C}_1(t) = 2\sqrt{p(1-p)}\,\mathrm{e}^{-\Gamma t} - 2p\,\left[\mathrm{e}^{-\Gamma t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\mathsf{C}_2(t) = -2\sqrt{\rho_{gg}(t)\rho_{ee}(t)} < 0$$

Can $\mathsf{C}_1(t) > 0$?

Independent atoms: $\Gamma_{12} = 0$

$$\boxed{\mathsf{C}_1(t) > 0} \quad \text{for} \quad t < t_d = \frac{1}{\Gamma}\ln\left(\frac{p + \sqrt{p(1-p)}}{2p-1}\right)$$

Independent atoms: $\Gamma_{12} = 0$

$$\boxed{\mathsf{C}_1(t) > 0} \quad \text{for} \quad t < t_d = \frac{1}{\Gamma} \ln\left(\frac{p + \sqrt{p(1-p)}}{2p-1}\right)$$

Death time t_d has finite value for $\boxed{p > 0.5}$

i.e. for population inversion: $\boxed{\rho_{ee}(0) > 0.5}$

Independent atoms: $\Gamma_{12} = 0$

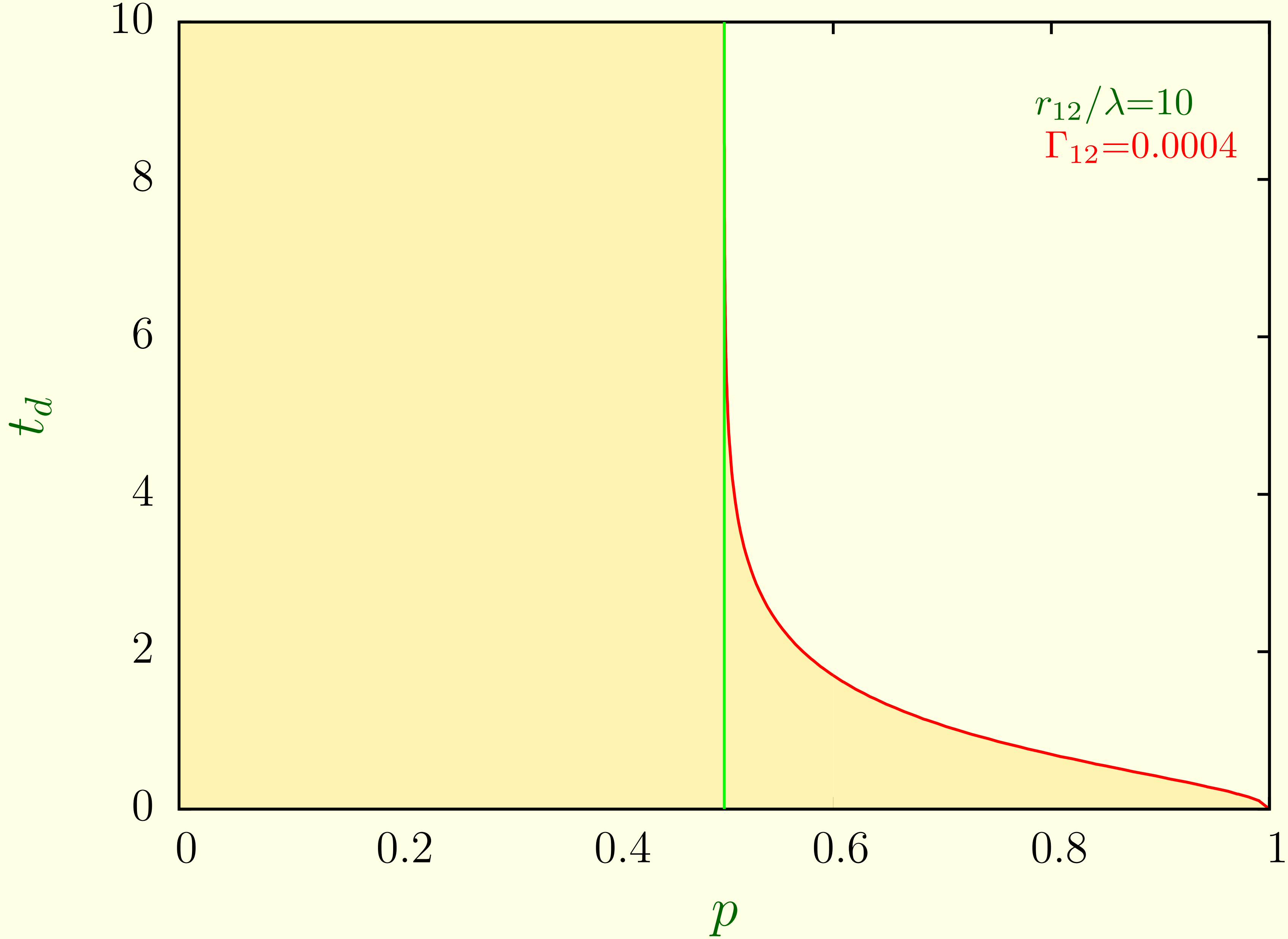
$$\boxed{\mathsf{C}_1(t) > 0} \quad \text{for} \quad t < t_d = \frac{1}{\Gamma} \ln\left(\frac{p + \sqrt{p(1-p)}}{2p-1}\right)$$

Death time t_d has finite value for $\boxed{p > 0.5}$

i.e. for population inversion: $\boxed{\rho_{ee}(0) > 0.5}$

Entanglement sudden death

Entanglement sudden death



Collective behavior: $\Gamma_{12} \neq 0$

$$\rho_{ee}(t) = p\,\mathrm{e}^{-2\Gamma t}$$

$$|\rho_{ge}(t)| = \sqrt{p(1-p)}\,\mathrm{e}^{-\Gamma t}$$

$$\rho_{ss}(t) = p\,\frac{\Gamma+\Gamma_{12}}{\Gamma-\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\rho_{aa}(t) = p\,\frac{\Gamma-\Gamma_{12}}{\Gamma+\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\mathsf{C}_1(t) = 2\,|\rho_{ge}(t)| - \left[\rho_{ss}(t) + \rho_{aa}(t)\right]$$

Collective behavior: $\Gamma_{12} \neq 0$

$$\rho_{ee}(t) = p\,\mathrm{e}^{-2\Gamma t}$$

$$|\rho_{ge}(t)| = \sqrt{p(1-p)}\,\mathrm{e}^{-\Gamma t}$$

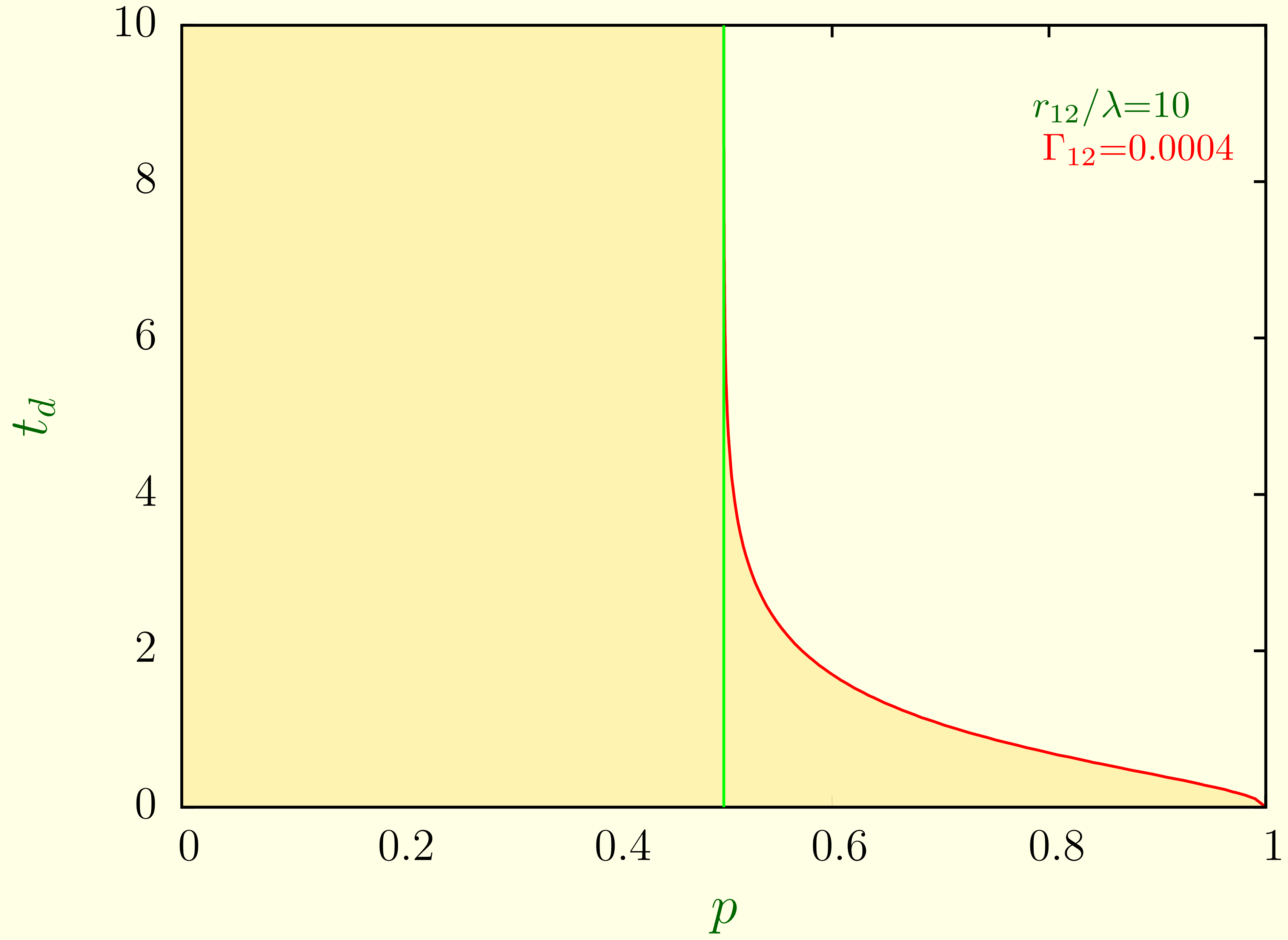
$$\rho_{ss}(t) = p\,\frac{\Gamma+\Gamma_{12}}{\Gamma-\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\rho_{aa}(t) = p\,\frac{\Gamma-\Gamma_{12}}{\Gamma+\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

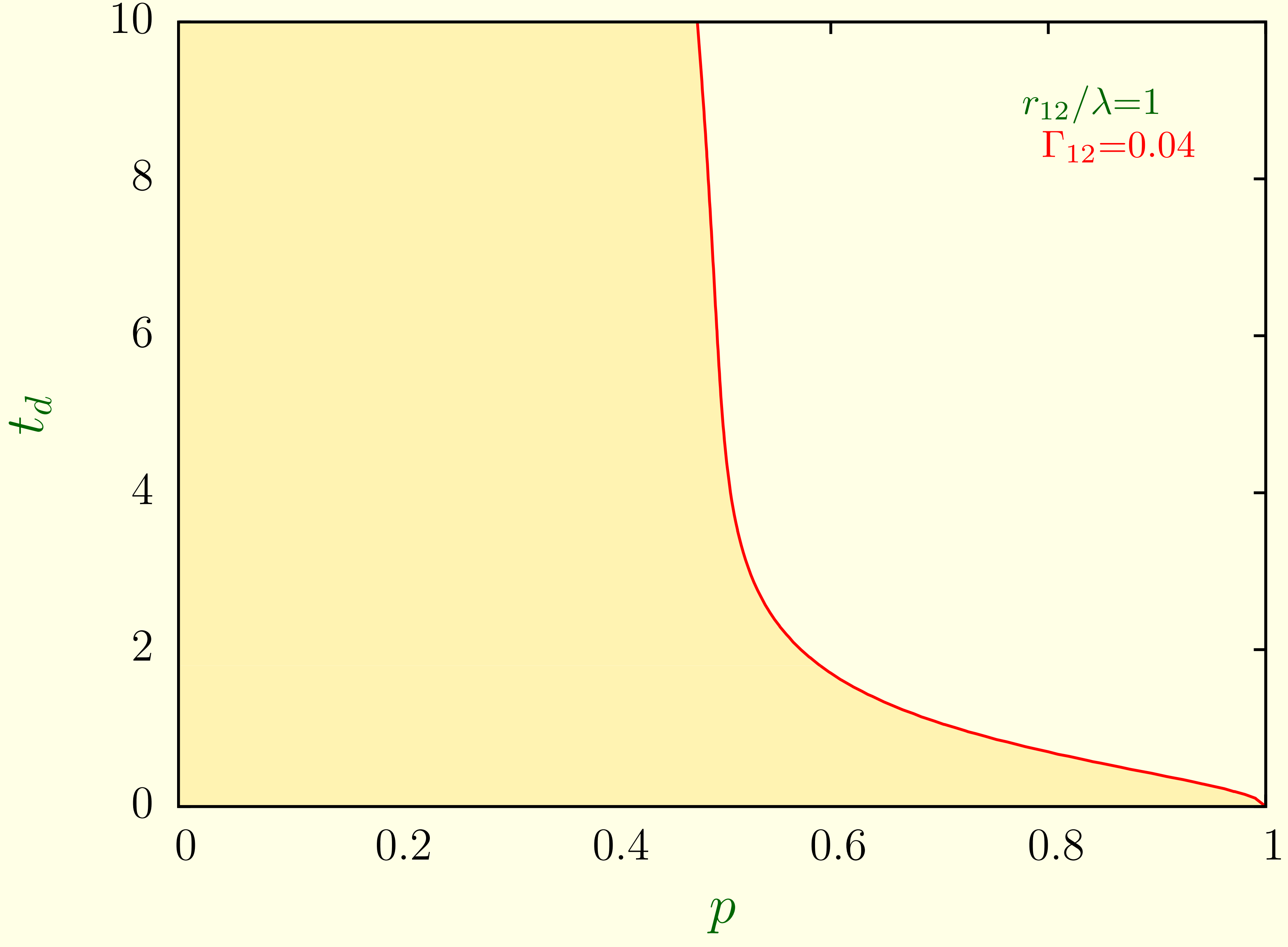
$$\mathsf{C}_1(t) = 2\,|\rho_{ge}(t)| - \left[\rho_{ss}(t) + \rho_{aa}(t)\right]$$

How about t_d?

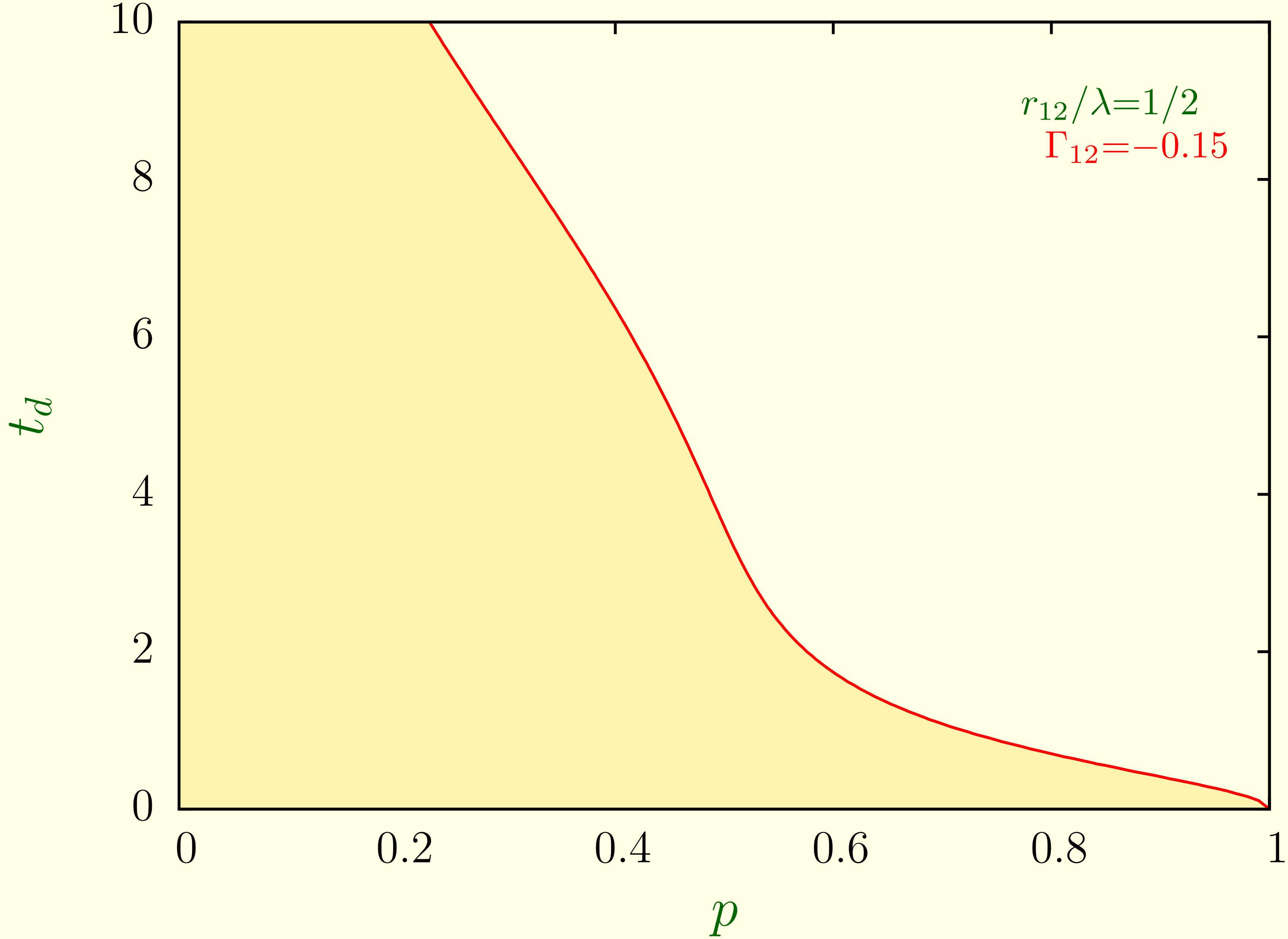
Entanglement sudden death



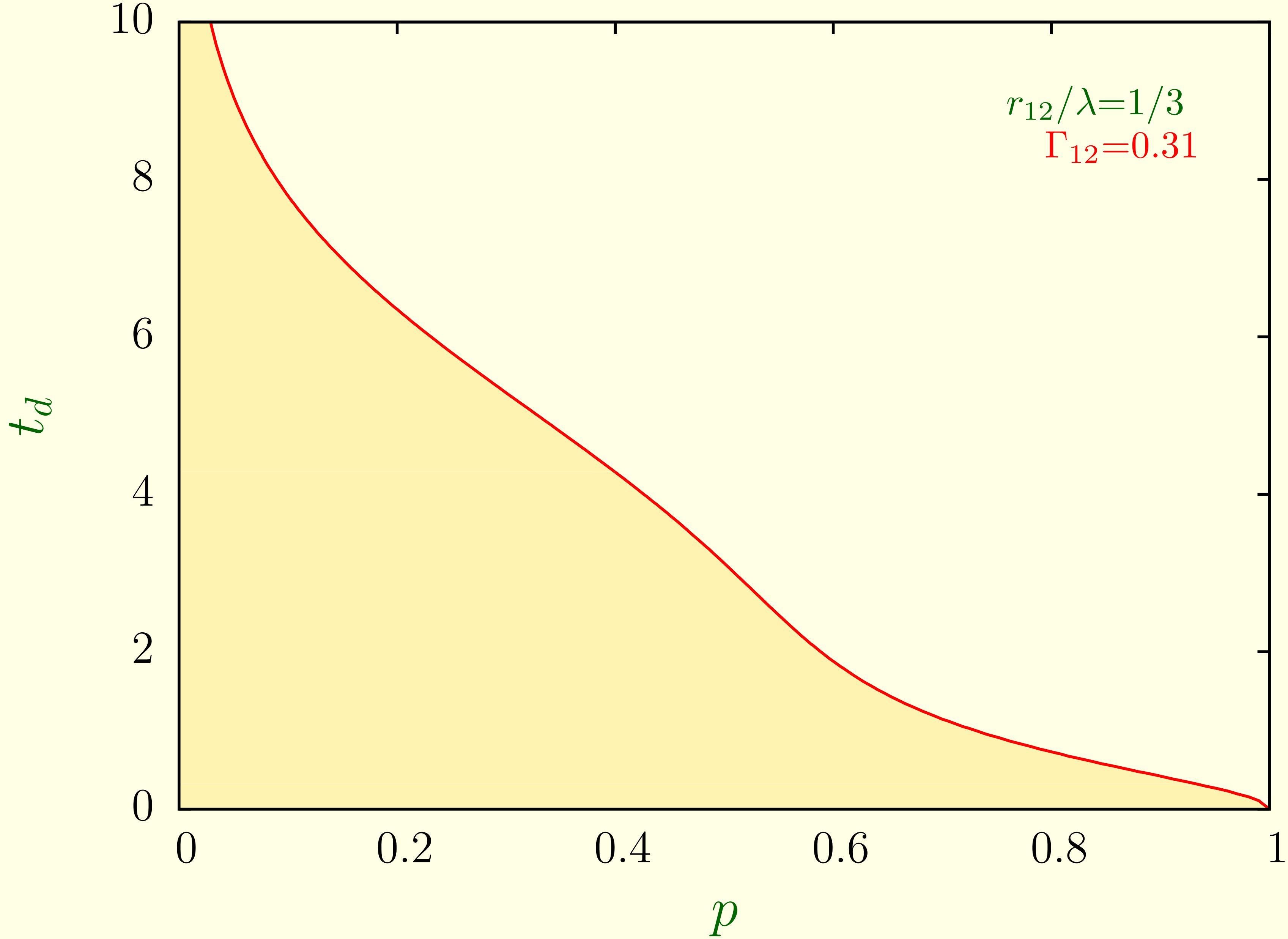
Entanglement sudden death

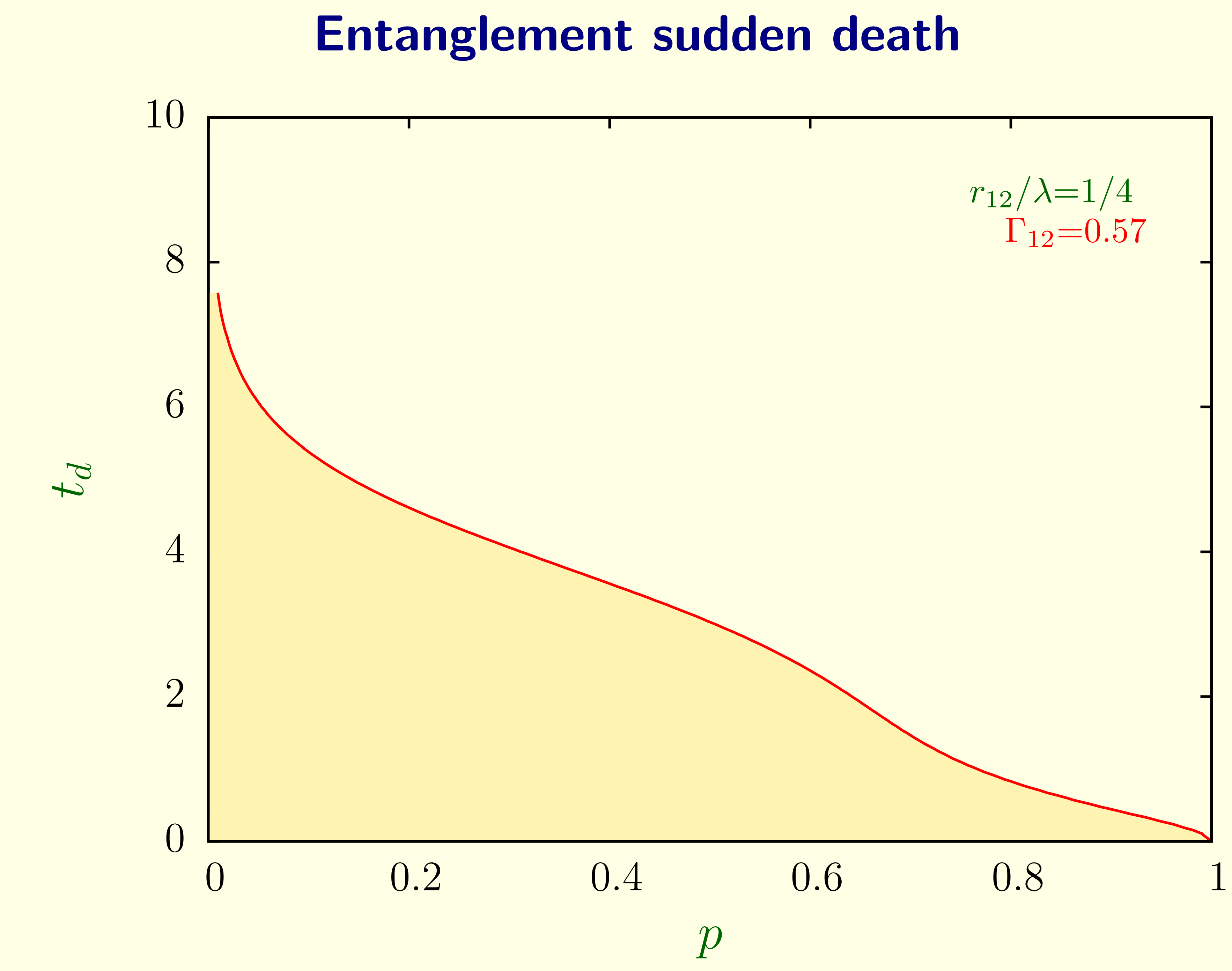


Entanglement sudden death

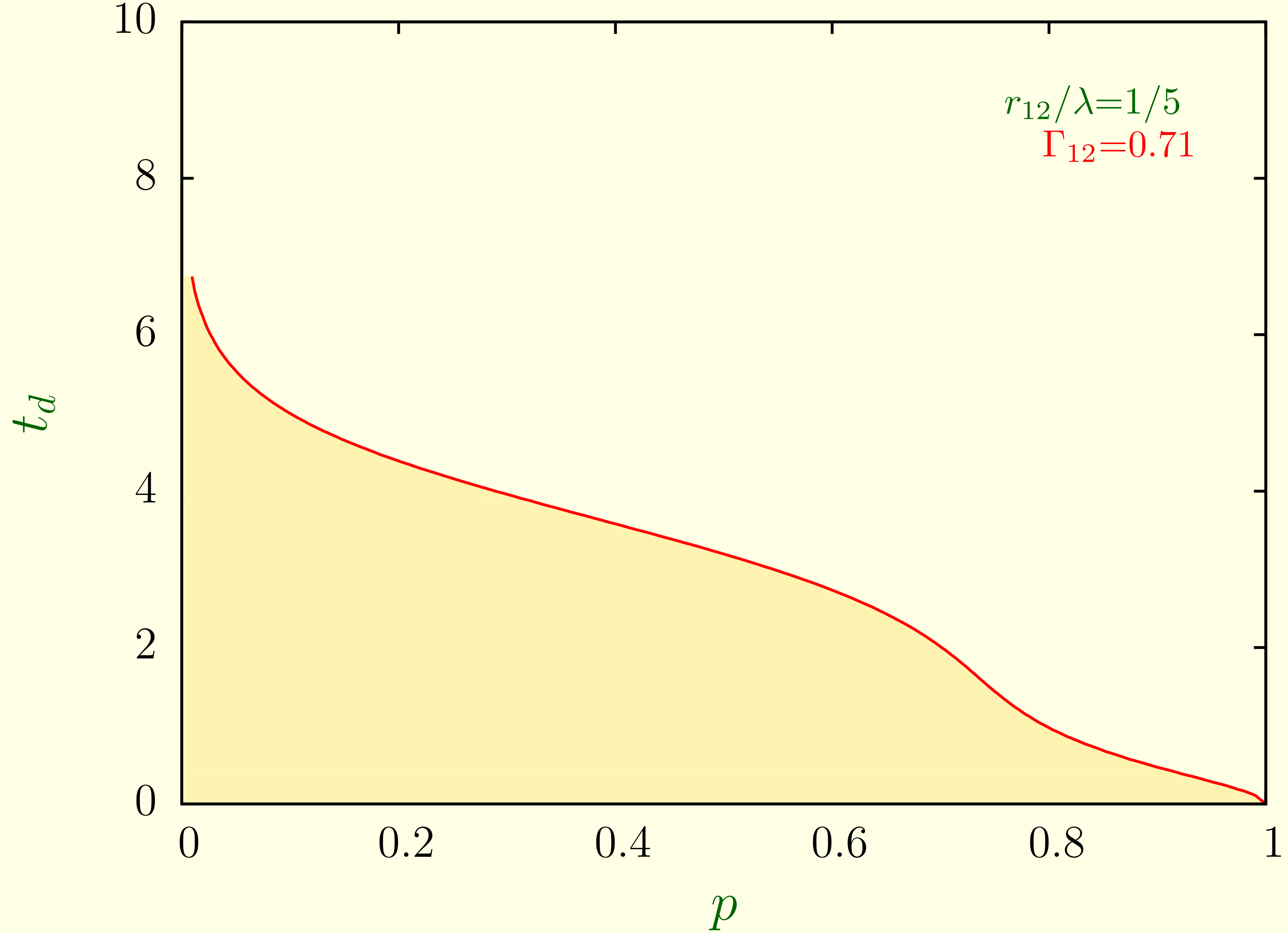


Entanglement sudden death

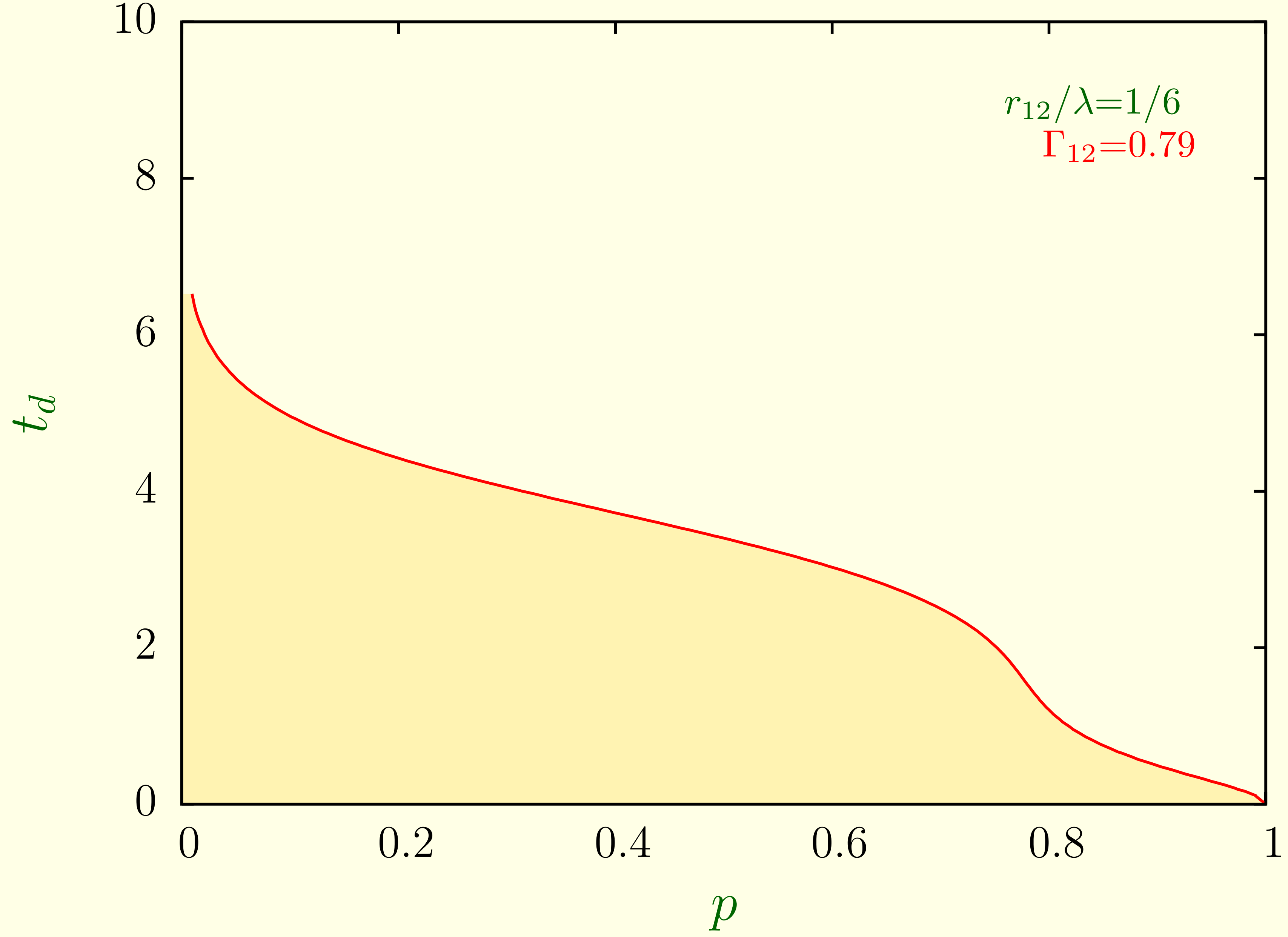



Entanglement sudden death
t_d
p
r_{12}/λ=1/4
Γ_{12}=0.57
0
2
4
6
8
10
0
0.2
0.4
0.6
0.8
1

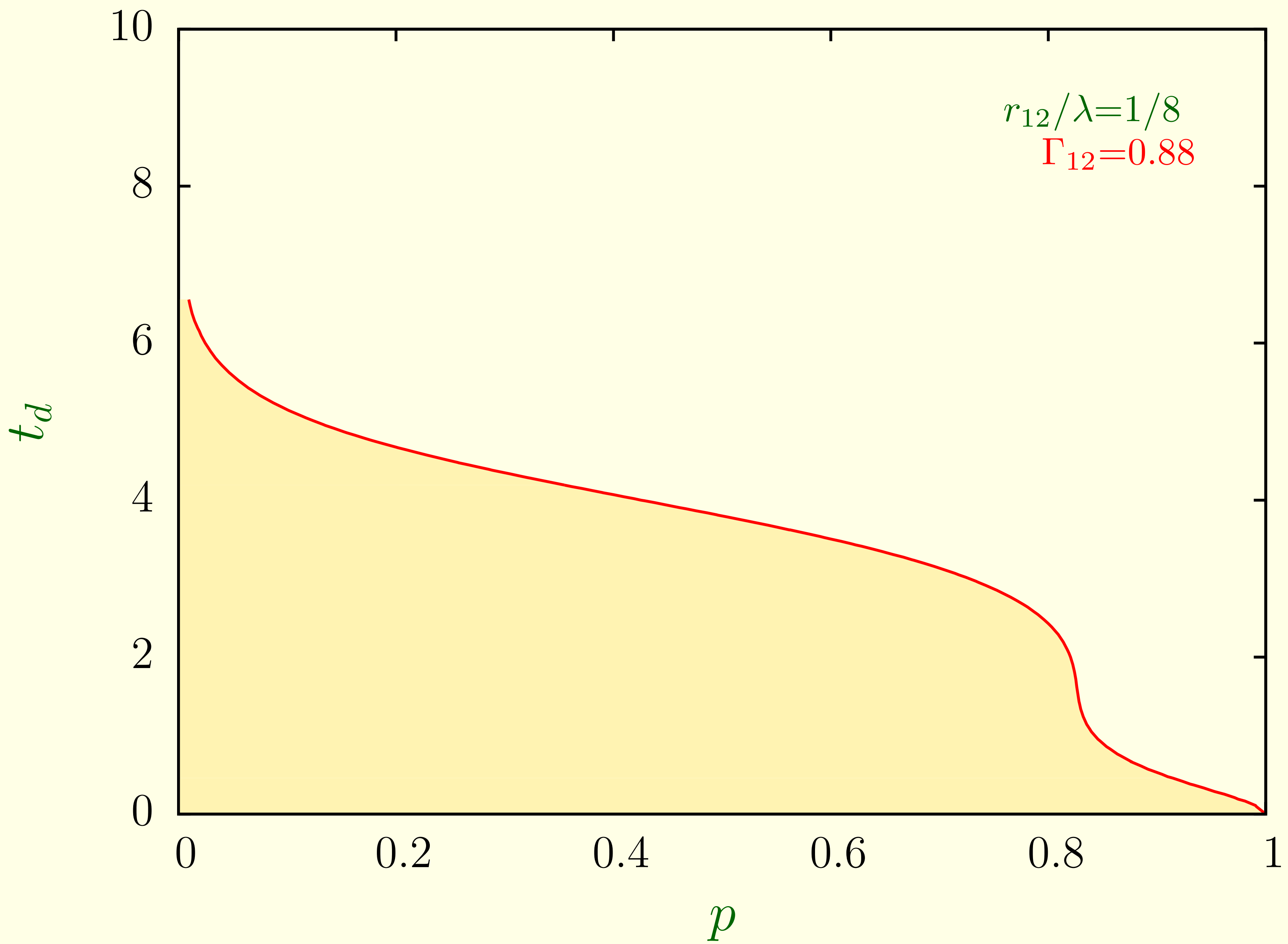
Entanglement sudden death



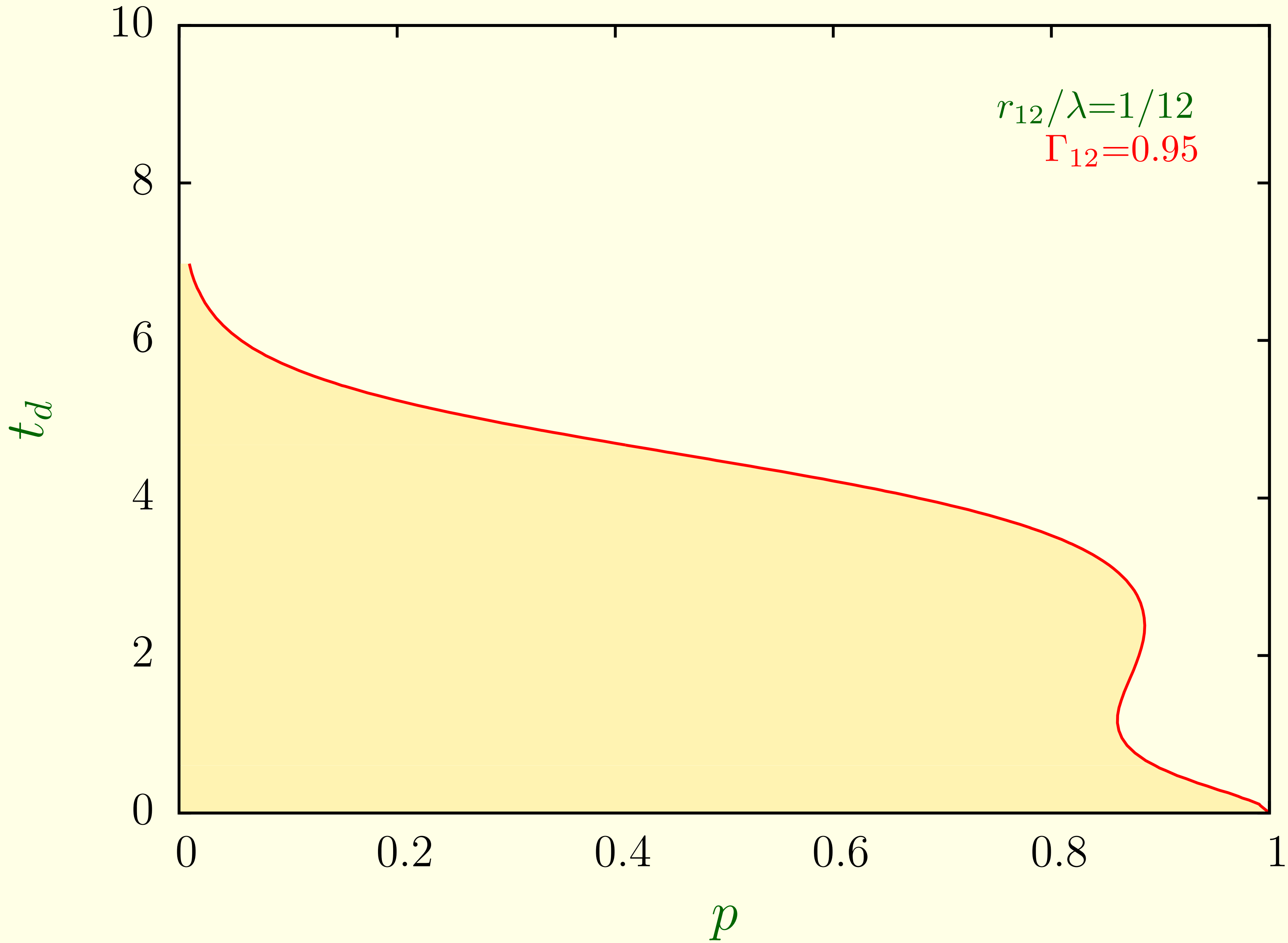
Entanglement sudden death



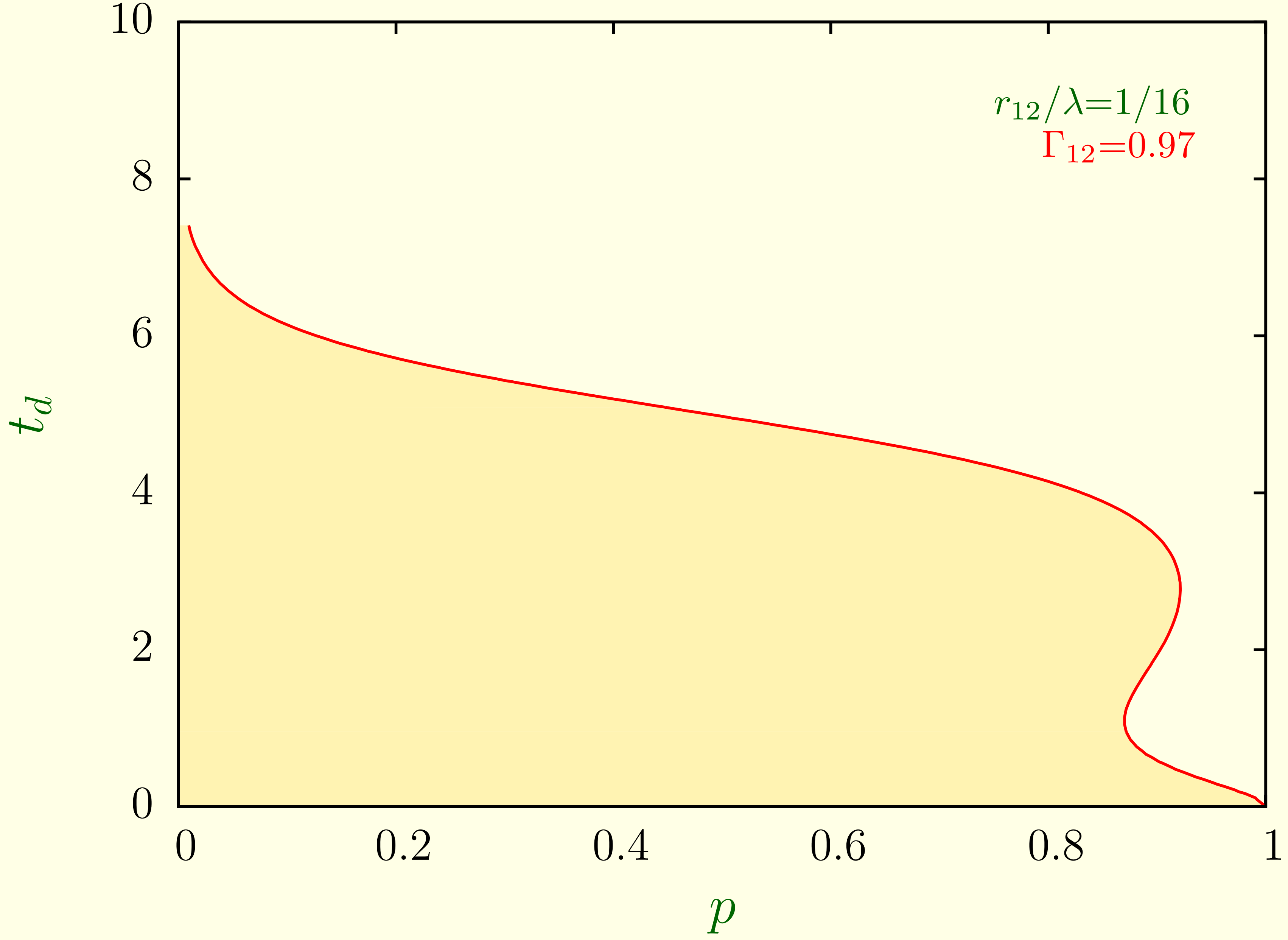
Entanglement sudden death



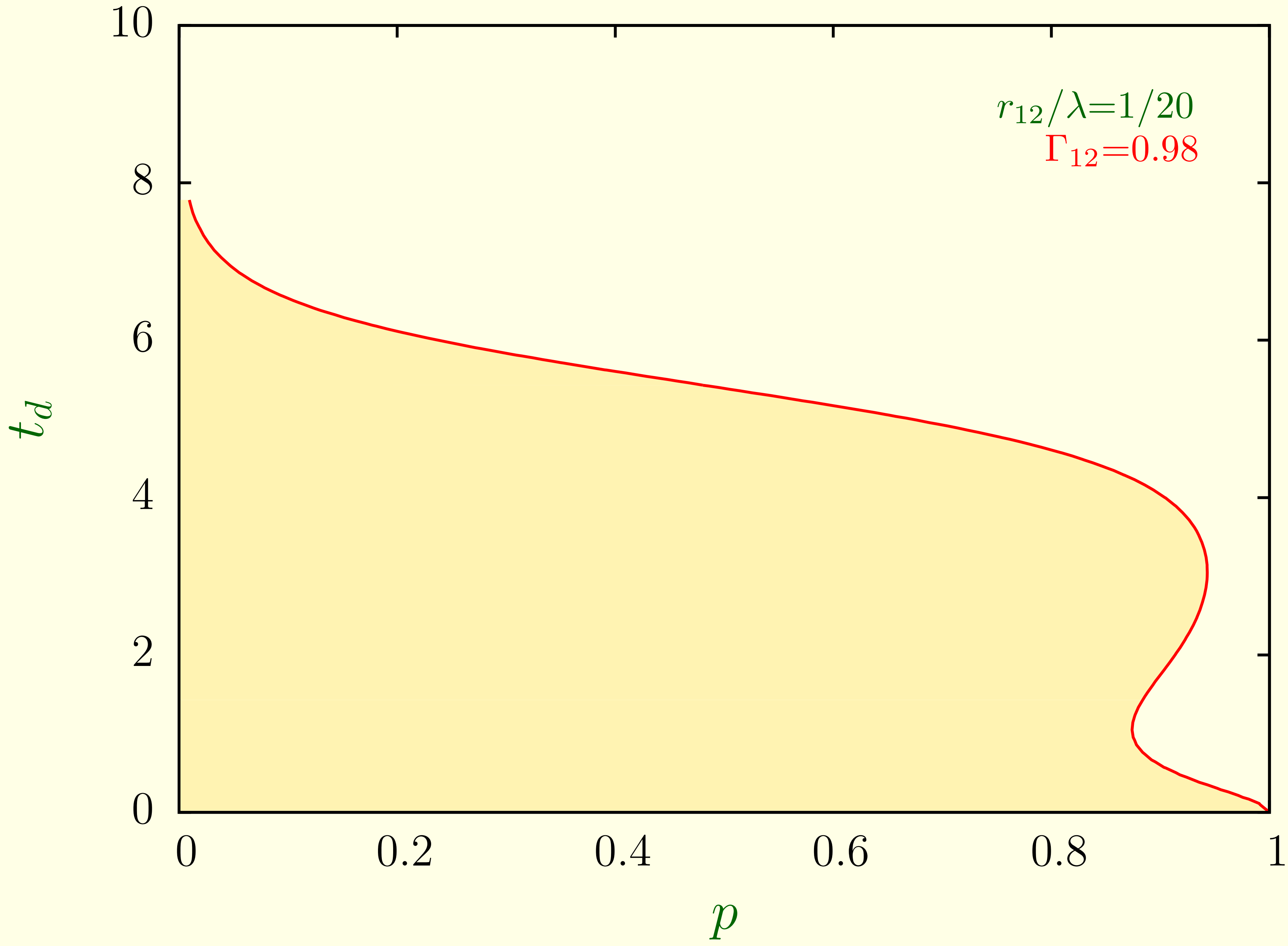
Entanglement sudden death



Entanglement sudden death



Entanglement sudden death



Collective behavior: $\Gamma_{12} \neq 0$

$$\rho_{ee}(t) = p\, \mathrm{e}^{-2\Gamma t}$$

$$|\rho_{ge}(t)| = \sqrt{p(1-p)}\, \mathrm{e}^{-\Gamma t}$$

$$\rho_{ss}(t) = p\, \frac{\Gamma+\Gamma_{12}}{\Gamma-\Gamma_{12}} \left[\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\rho_{aa}(t) = p\, \frac{\Gamma-\Gamma_{12}}{\Gamma+\Gamma_{12}} \left[\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

Collective behavior: $\Gamma_{12} \neq 0$

$$\rho_{ee}(t) = p\,\mathrm{e}^{-2\Gamma t}$$

$$|\rho_{ge}(t)| = \sqrt{p(1-p)}\,\mathrm{e}^{-\Gamma t}$$

$$\rho_{ss}(t) = p\,\frac{\Gamma+\Gamma_{12}}{\Gamma-\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\rho_{aa}(t) = p\,\frac{\Gamma-\Gamma_{12}}{\Gamma+\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\rho_{ss} \neq \rho_{aa}$$

$$\mathsf{C}_2(t) = |\rho_{ss}(t) - \rho_{aa}(t)| - 2\sqrt{\rho_{gg}(t)\rho_{ee}(t)}$$

Collective behavior: $\Gamma_{12} \neq 0$

$$\rho_{ee}(t) = p\,\mathrm{e}^{-2\Gamma t}$$

$$|\rho_{ge}(t)| = \sqrt{p(1-p)}\,\mathrm{e}^{-\Gamma t}$$

$$\rho_{ss}(t) = p\,\frac{\Gamma+\Gamma_{12}}{\Gamma-\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\rho_{aa}(t) = p\,\frac{\Gamma-\Gamma_{12}}{\Gamma+\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\rho_{ss} \neq \rho_{aa}$$

$$\mathsf{C}_2(t) = |\rho_{ss}(t) - \rho_{aa}(t)| - 2\sqrt{\rho_{gg}(t)\rho_{ee}(t)}$$

Can $\mathsf{C}_2(t)$ become positive for some $t_r > t_d$?

Collective behavior: $\Gamma_{12} \neq 0$

$$\rho_{ee}(t) = p\,\mathrm{e}^{-2\Gamma t}$$

$$|\rho_{ge}(t)| = \sqrt{p(1-p)}\,\mathrm{e}^{-\Gamma t}$$

$$\rho_{ss}(t) = p\,\frac{\Gamma+\Gamma_{12}}{\Gamma-\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

$$\rho_{aa}(t) = p\,\frac{\Gamma-\Gamma_{12}}{\Gamma+\Gamma_{12}}\left[\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t}\right]$$

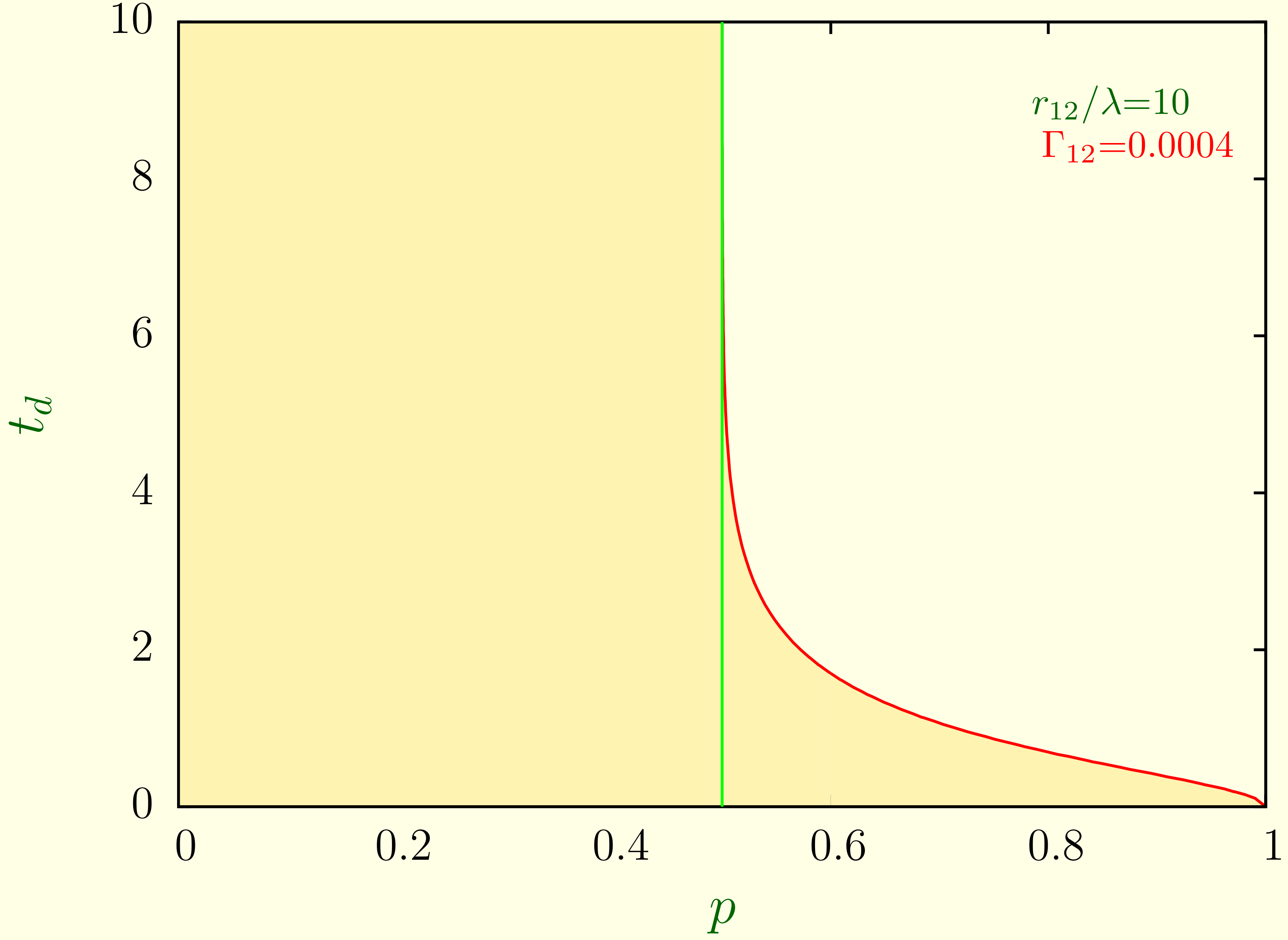
$$\rho_{ss} \neq \rho_{aa}$$

$$\mathsf{C}_2(t) = |\rho_{ss}(t) - \rho_{aa}(t)| - 2\sqrt{\rho_{gg}(t)\rho_{ee}(t)}$$

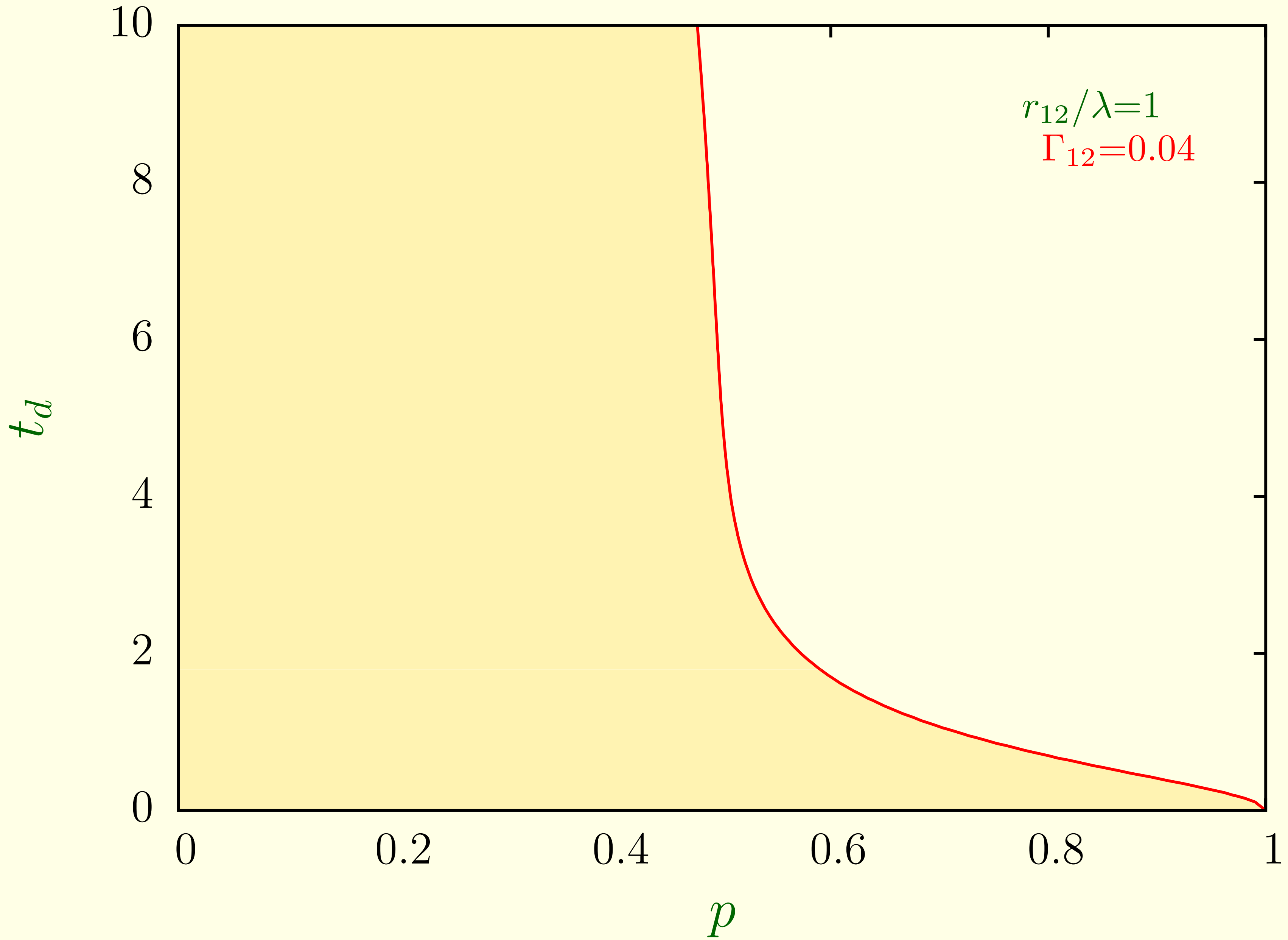
Can $\mathsf{C}_2(t)$ become positive for some $t_r > t_d$?

Can entanglement revive?

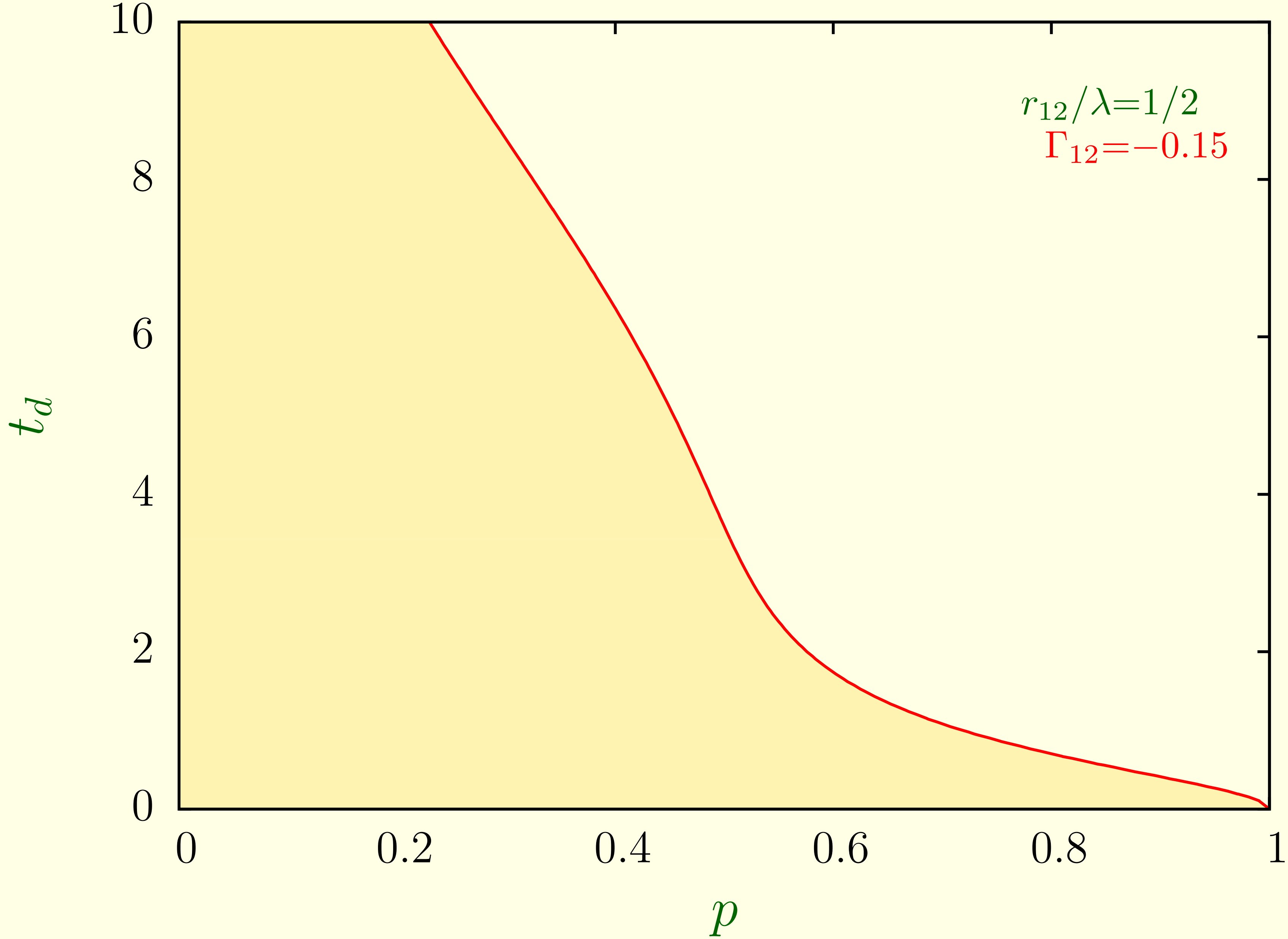
Entanglement sudden death



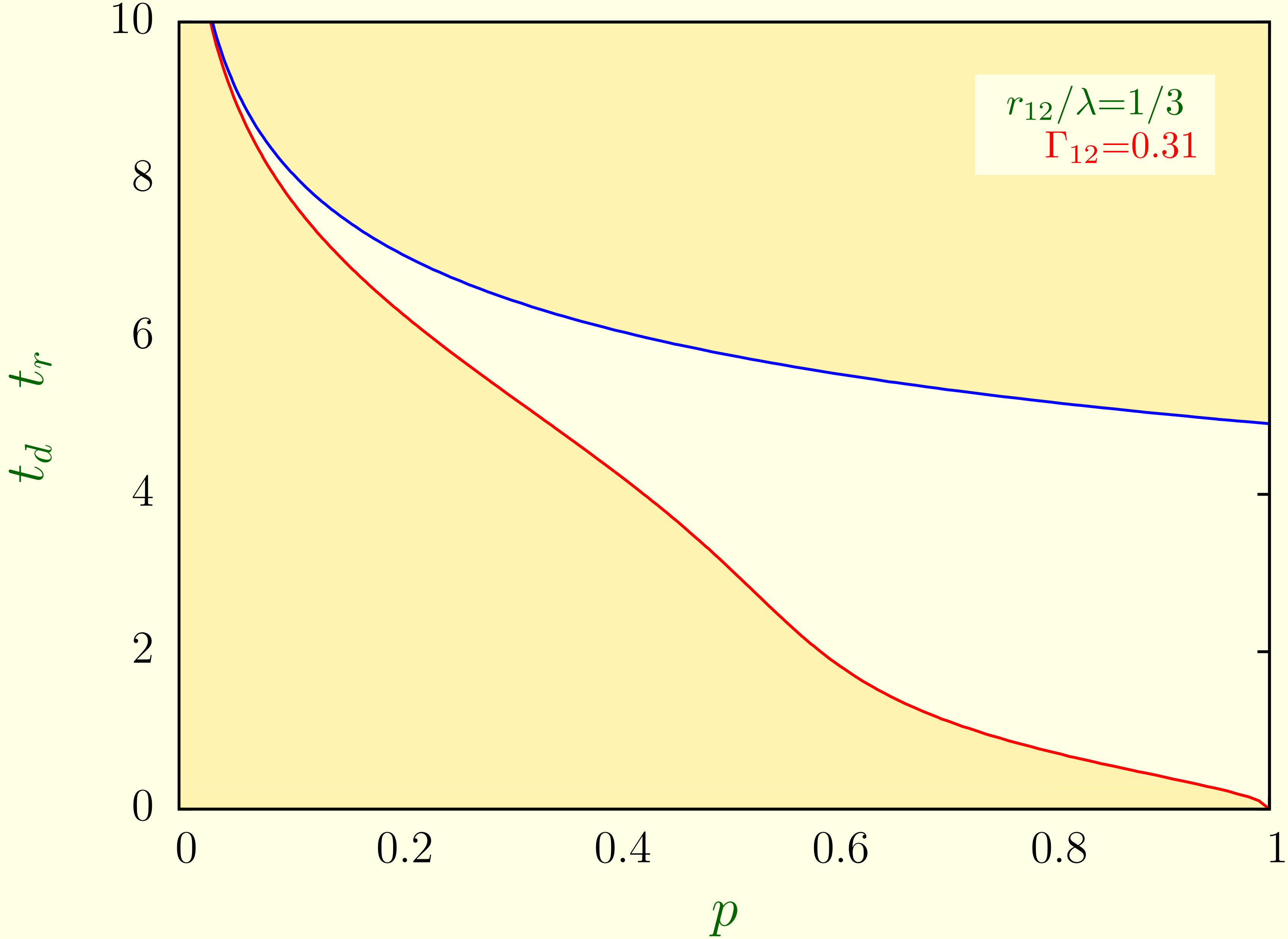
Entanglement sudden death



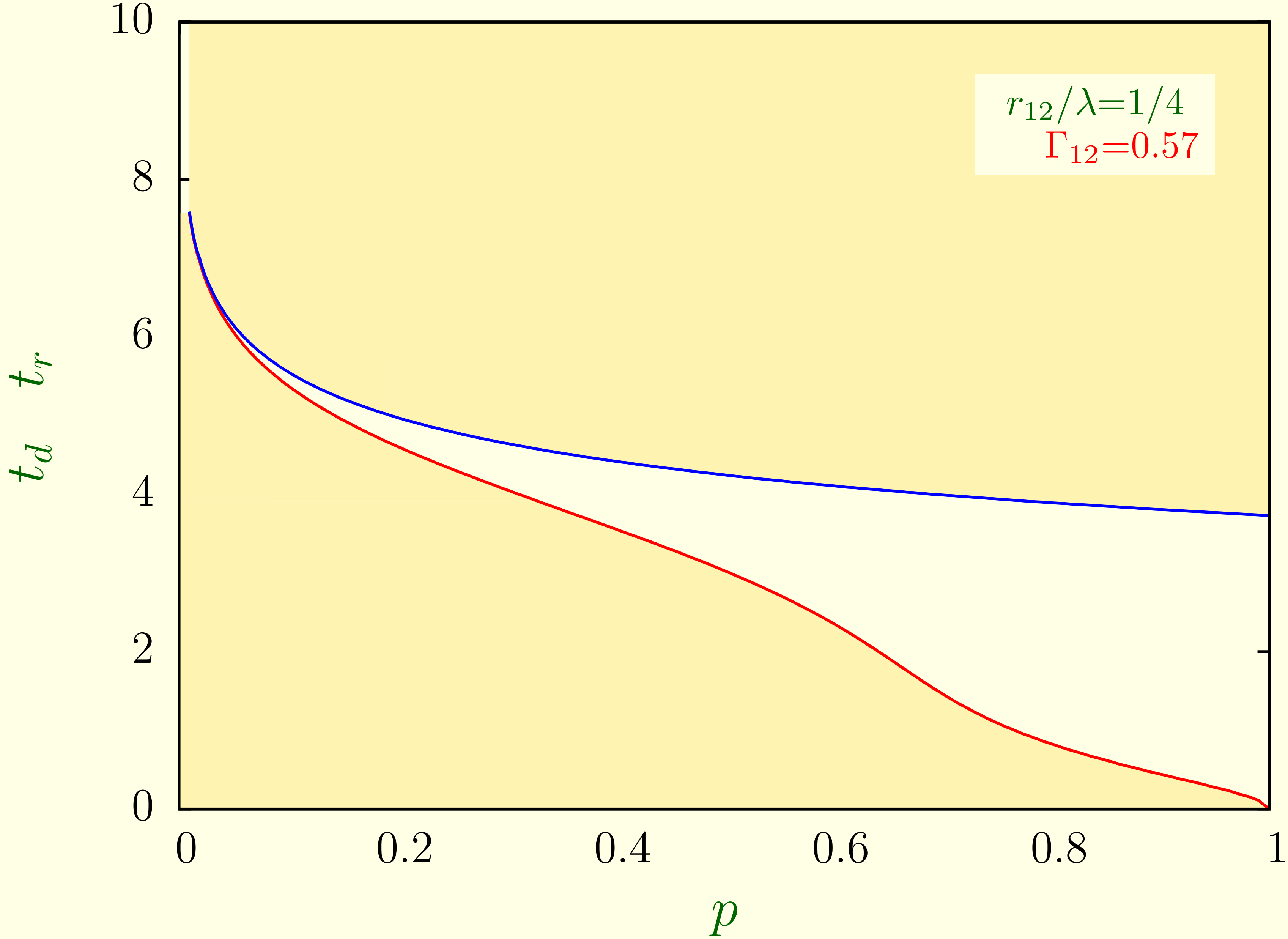
Entanglement sudden death



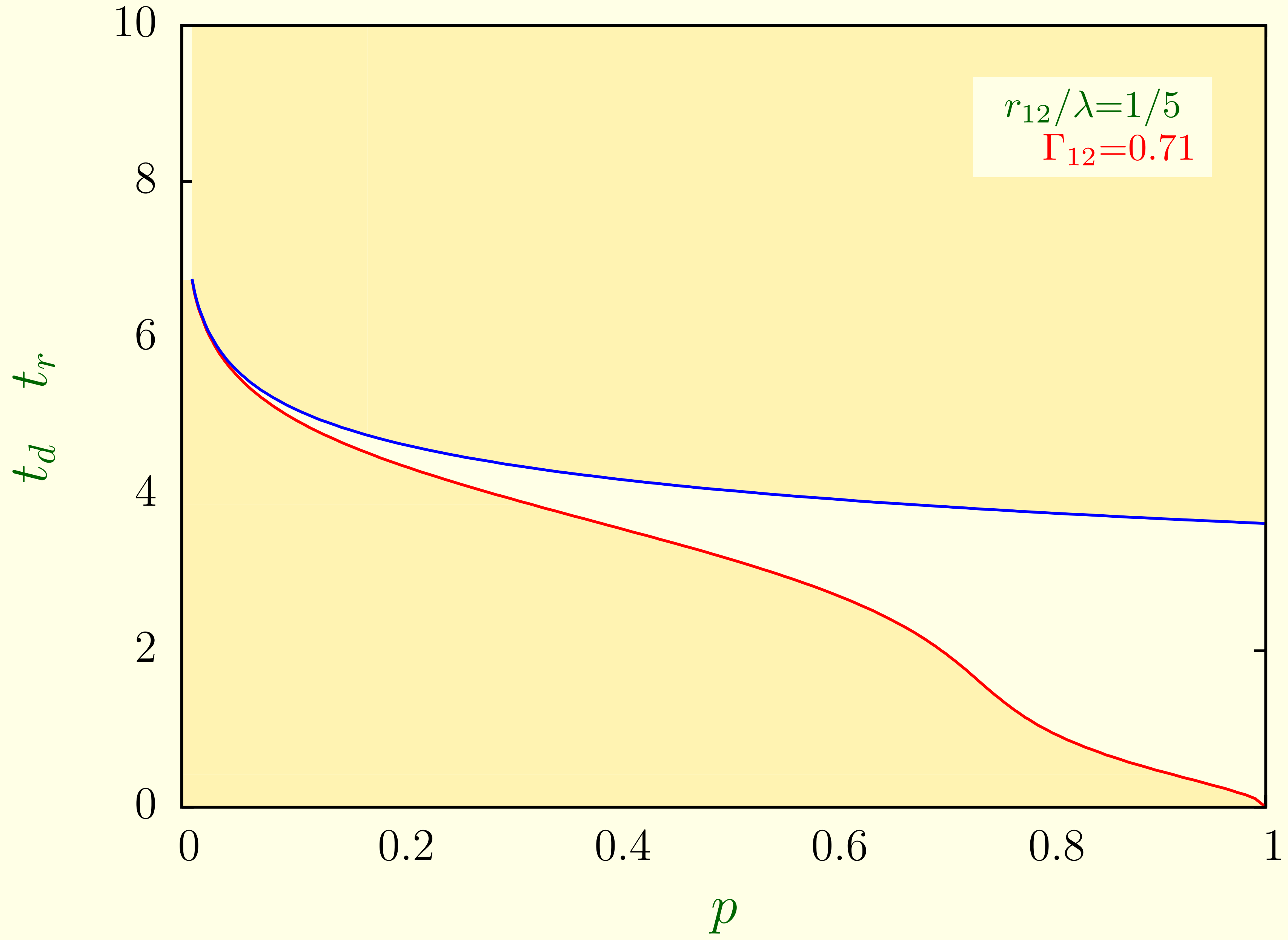
Entanglement sudden death and revival



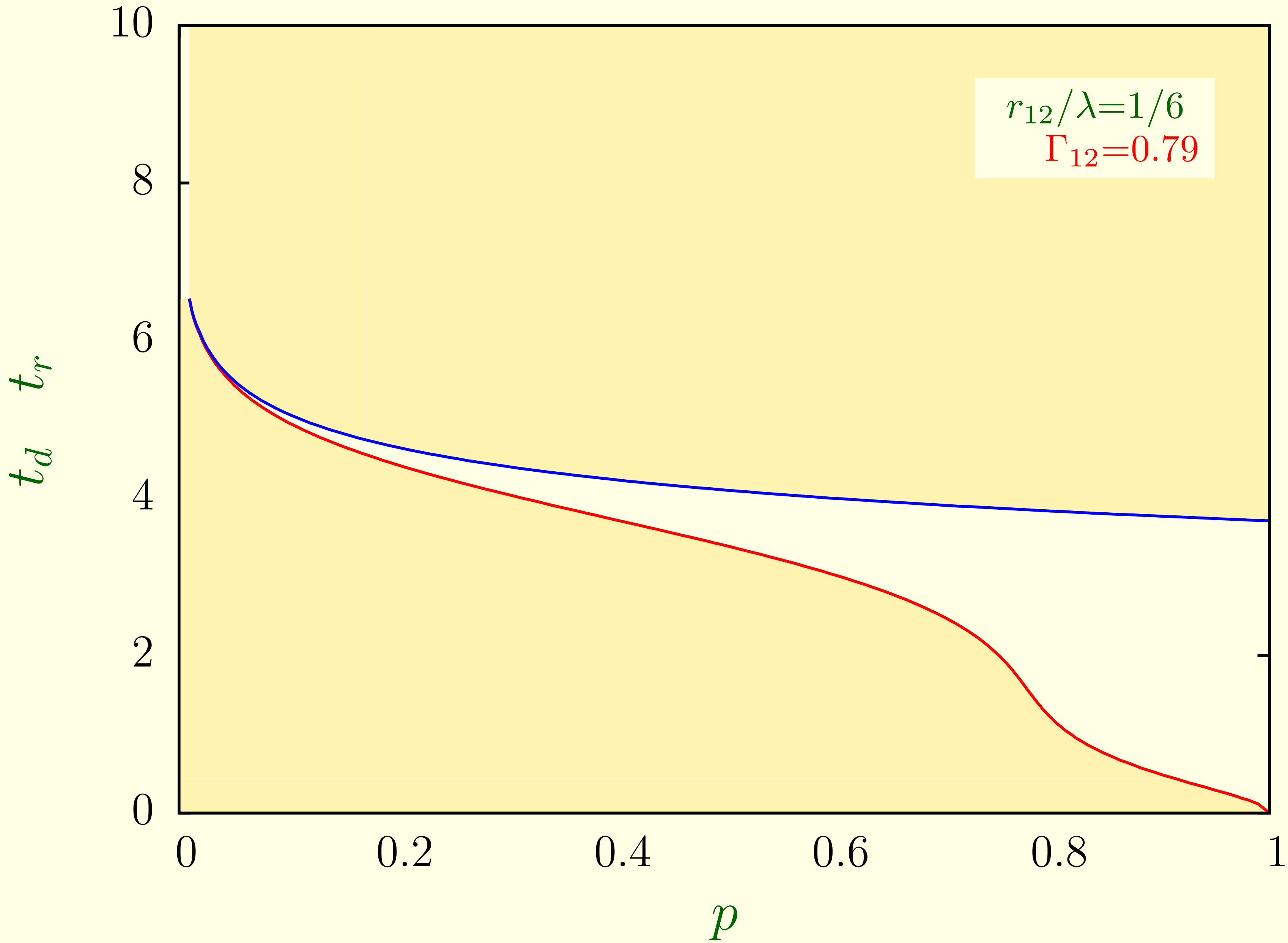
Entanglement sudden death and revival



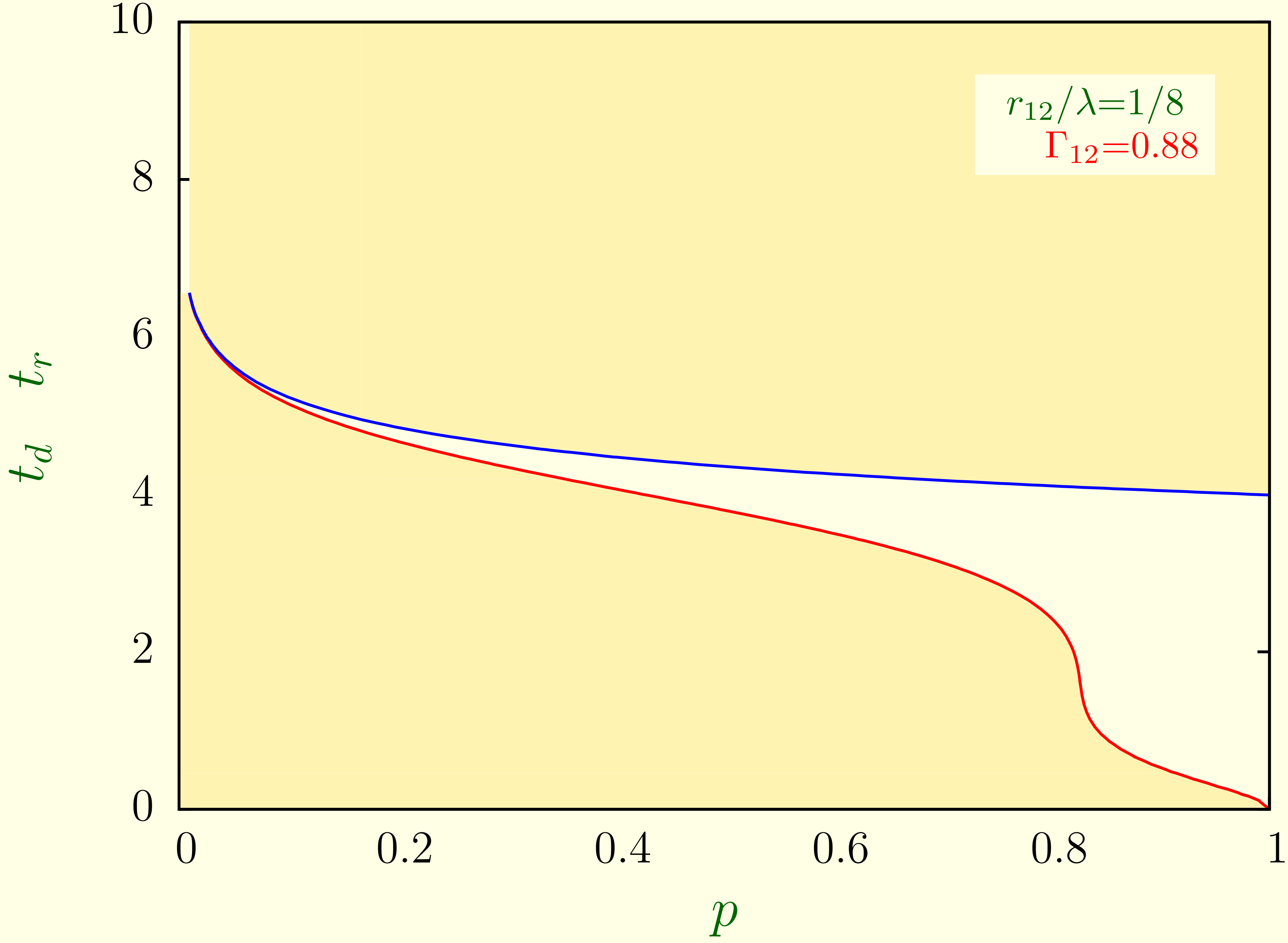
Entanglement sudden death and revival



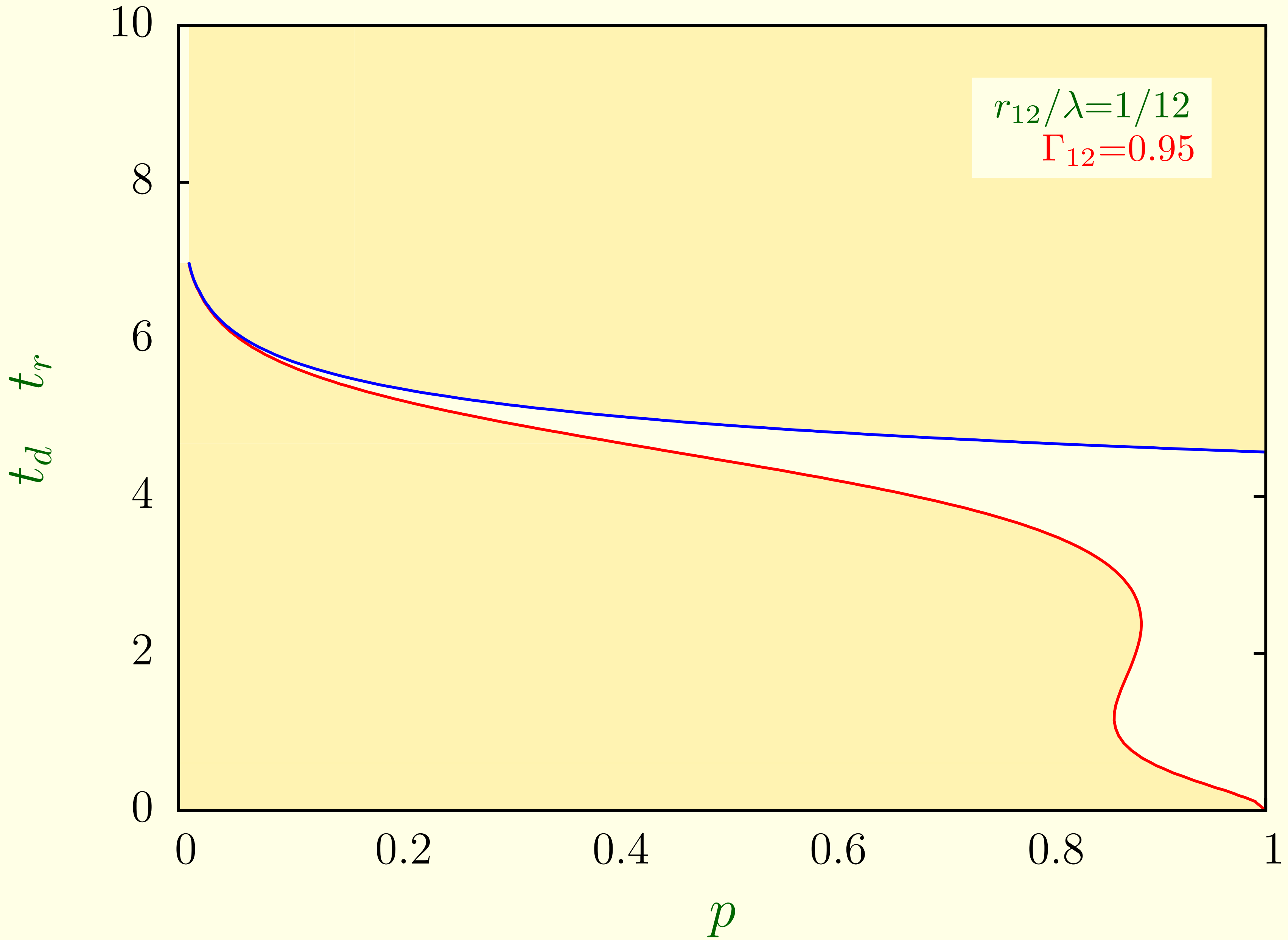
Entanglement sudden death and revival



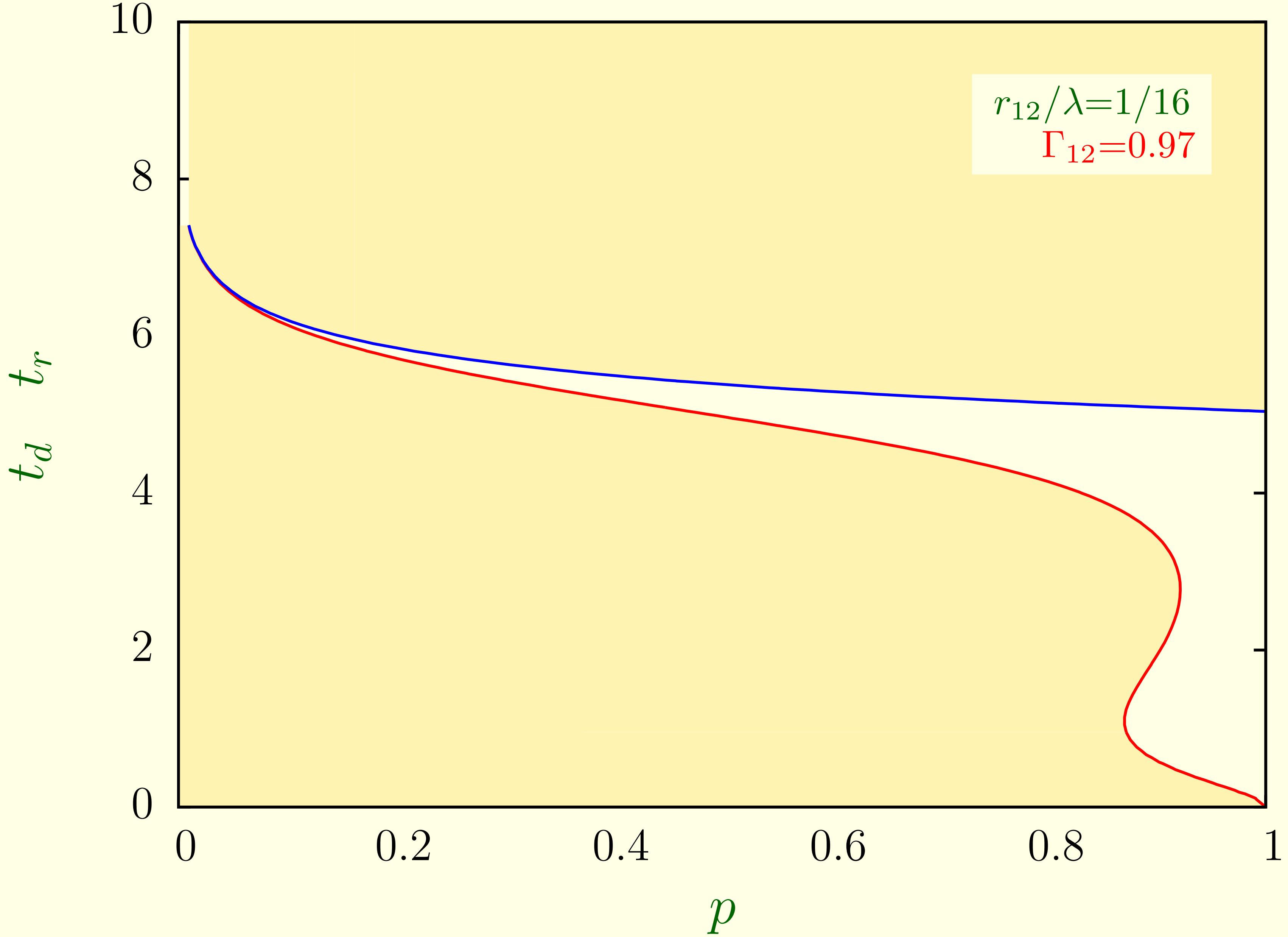
Entanglement sudden death and revival



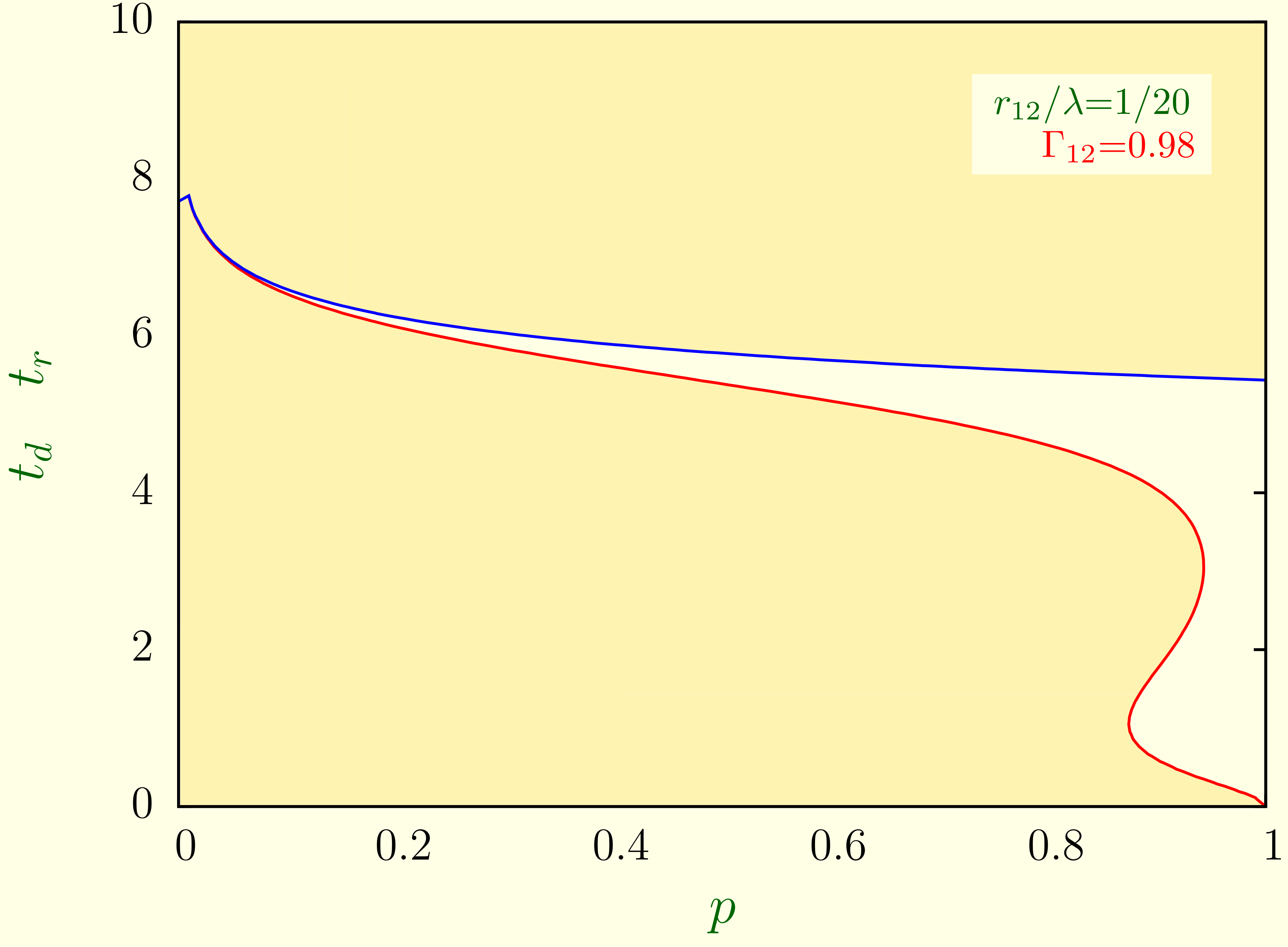
Entanglement sudden death and revival



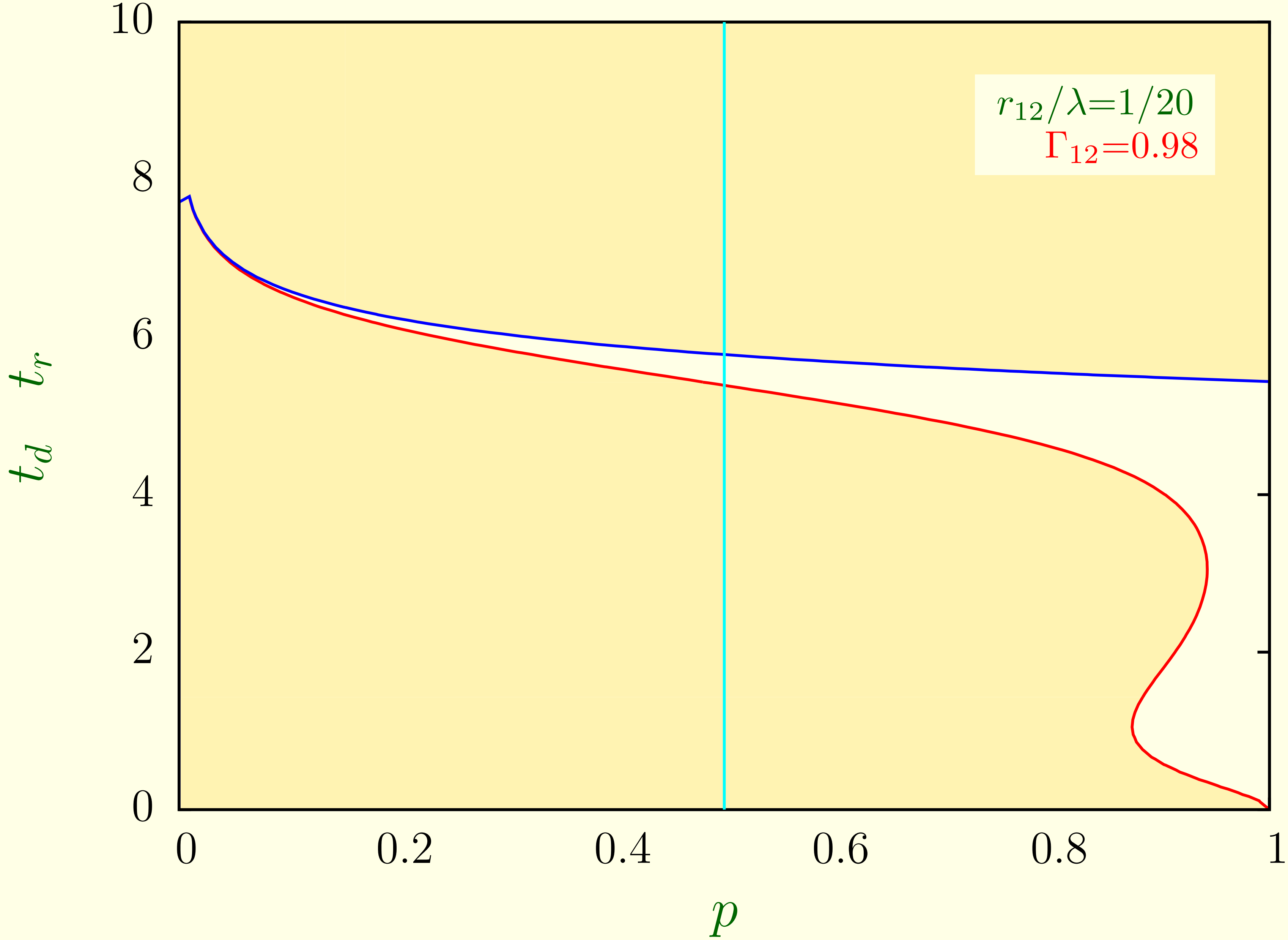
Entanglement sudden death and revival



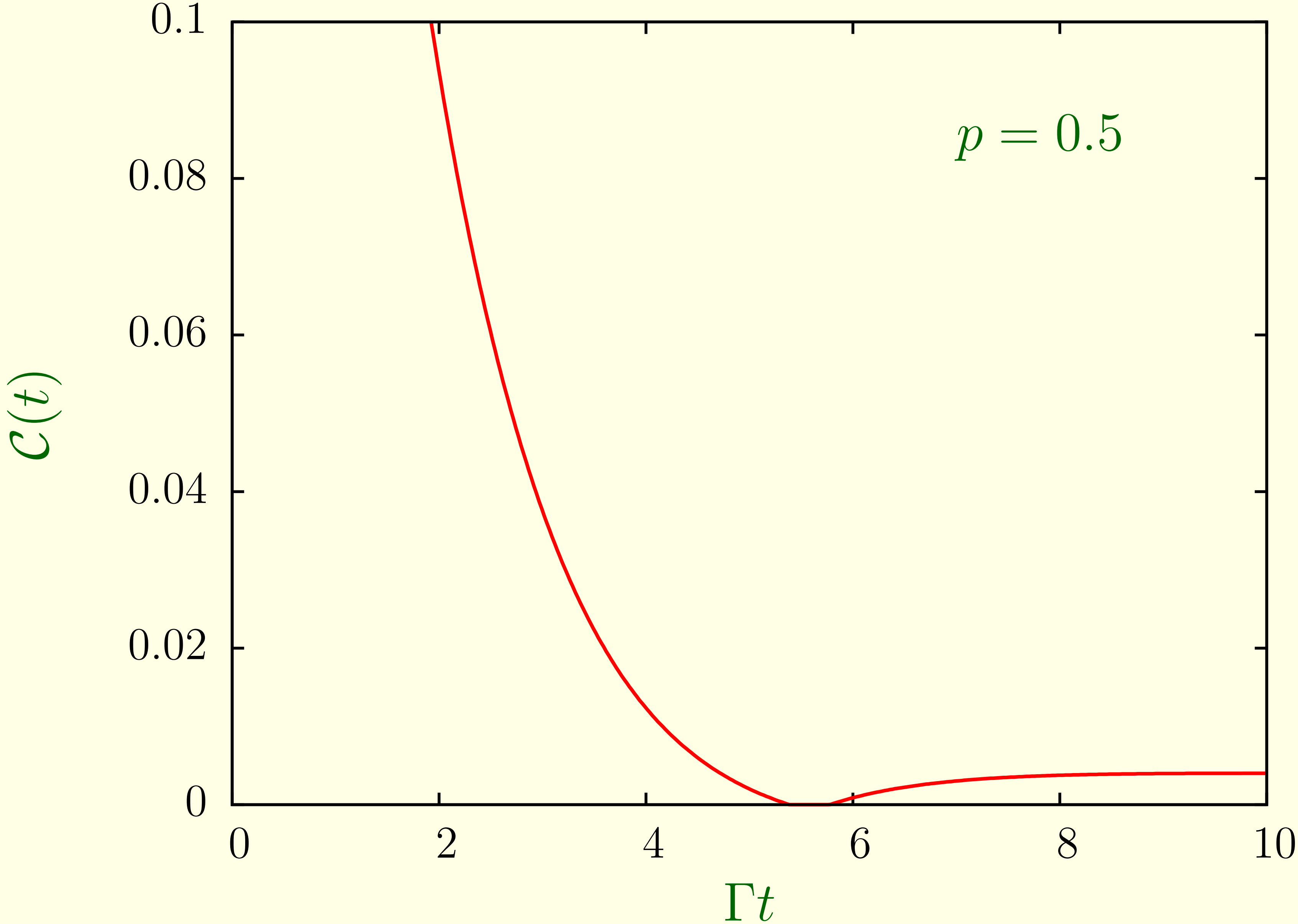
Entanglement sudden death and revival



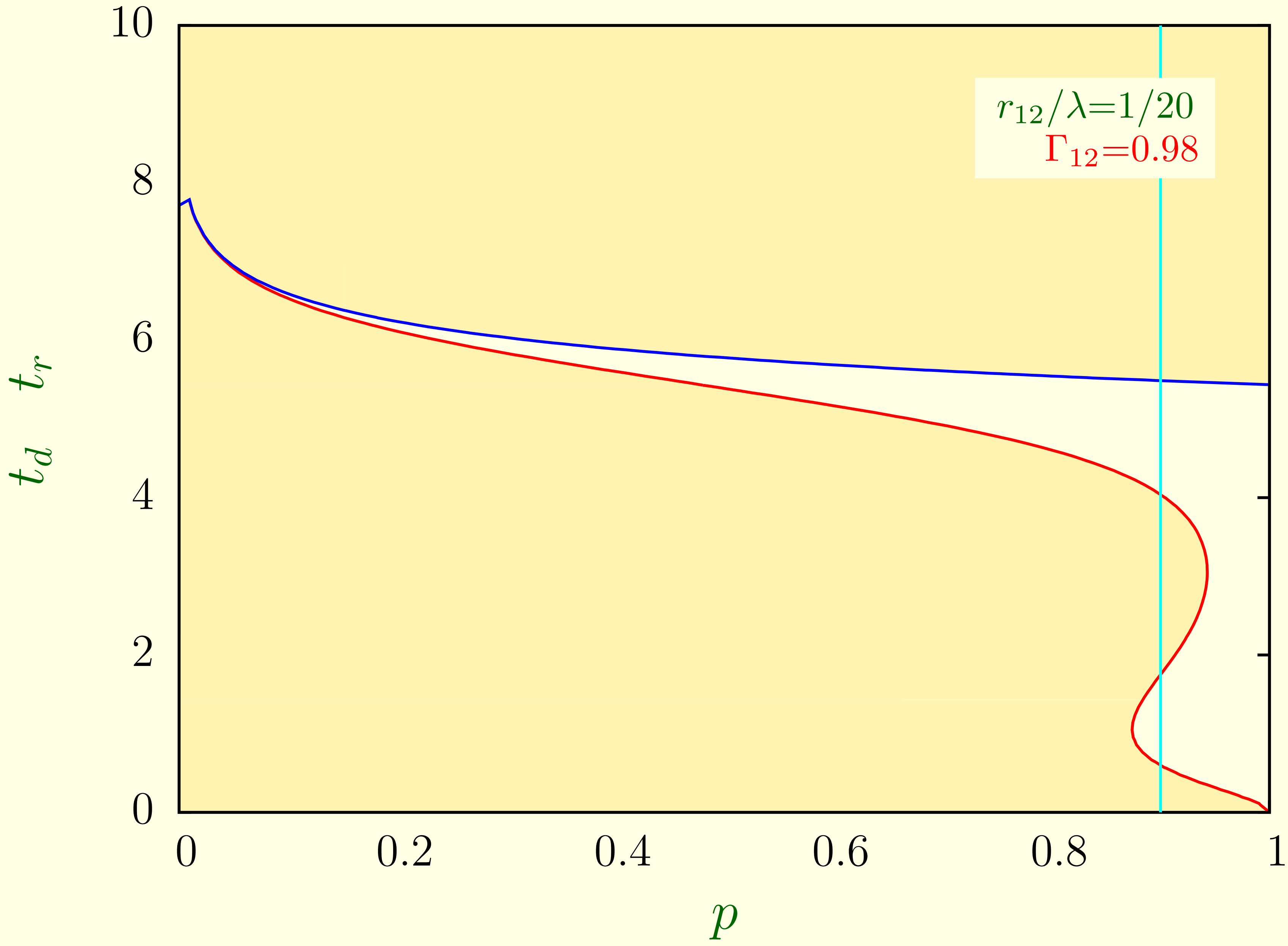
Example: $p = 0.5$



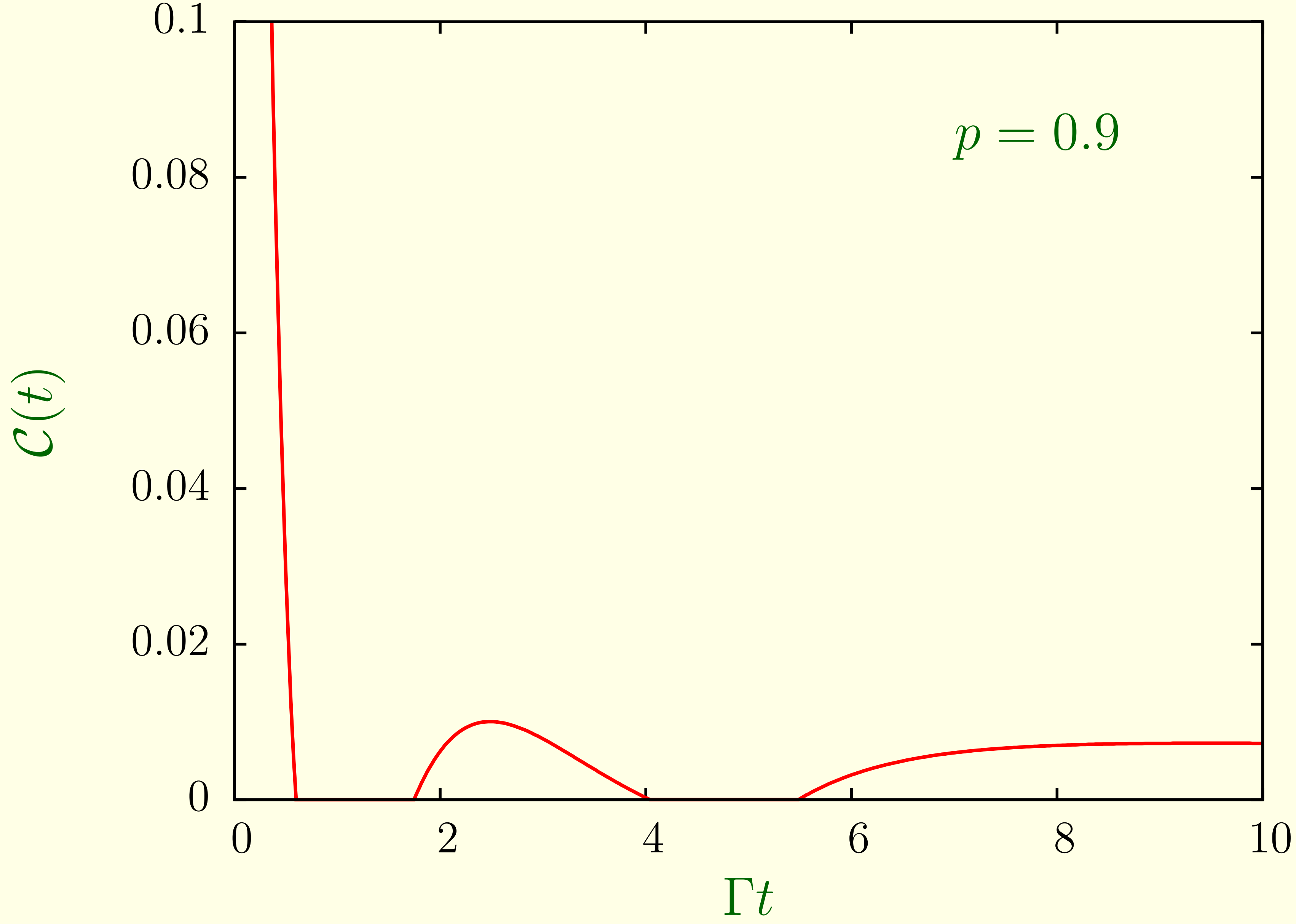
Example: $p = 0.5$



Example: $p = 0.9$



Example: $p = 0.9$



After sudden death . . .

After sudden death . . .

. . . comes revival . . .

After sudden death . . .

. . . comes revival . . .

. . . if two atoms behave collectively!

3.3 Birth of entanglement

R. Tanaś, Z. Ficek, J. Opt. B **6**, S90 (2004)

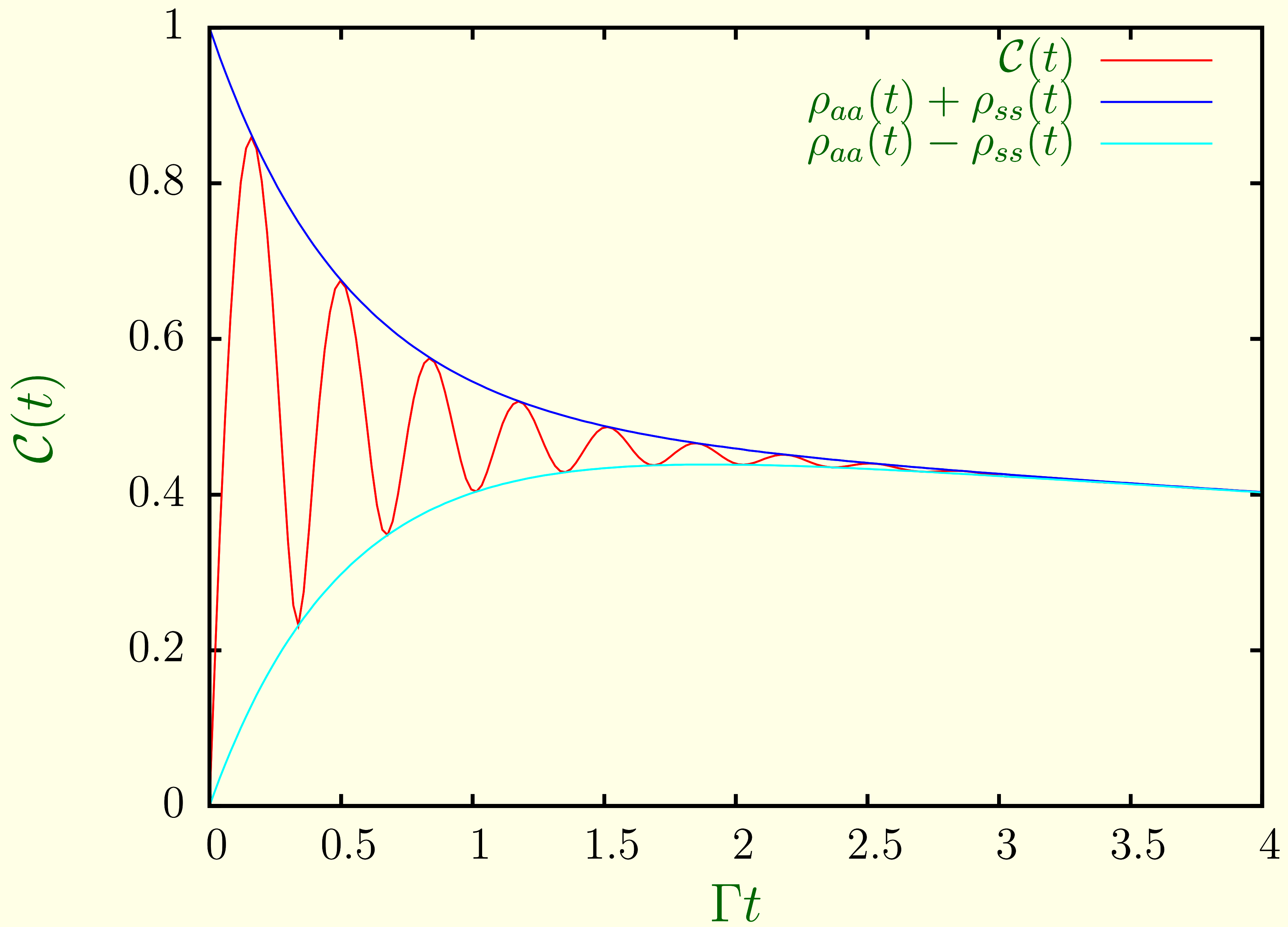
Initial state: $|\Psi(0)\rangle = |e_1\rangle \otimes |g_2\rangle$ (one atom excited)

No initial entanglement

Concurrence:

$$\mathcal{C}(t) = \frac{1}{2}\sqrt{\left[\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-(\Gamma-\Gamma_{12})t}\right]^2 + \left[2\mathrm{e}^{-\Gamma t}\sin(2\Omega_{12}t)\right]^2}$$

Smooth birth of entanglement: $\mathcal{C}(t > 0) > 0$



Concurrence $\mathcal{C}(t)$ for $|\Psi(0)\rangle = |e_1\rangle \otimes |g_2\rangle$

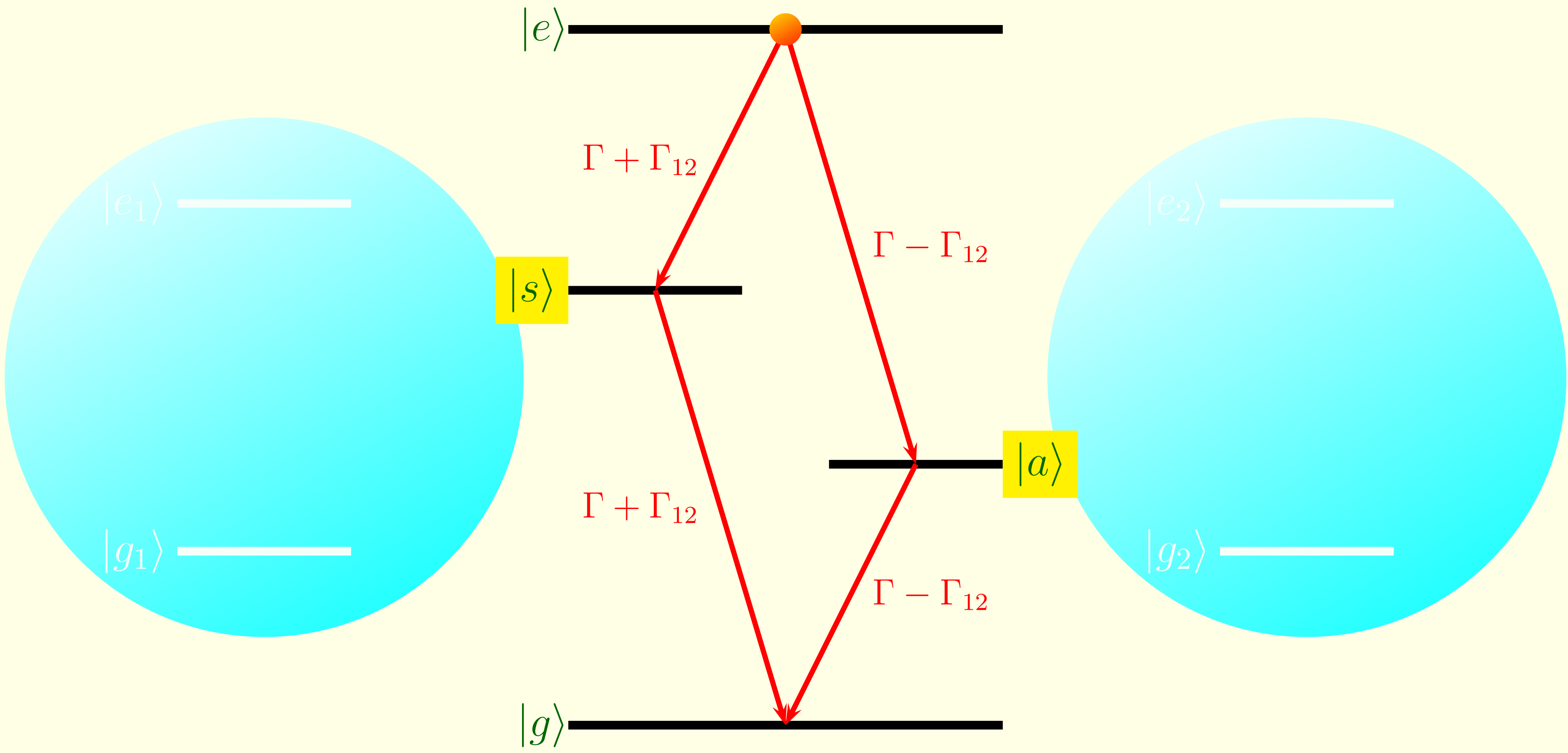
$(\hat{\boldsymbol{\mu}} \perp \hat{\boldsymbol{r}}_{12},\ r_{12} = \lambda/12,\ \Gamma_{12} = 0.95\,\Gamma,\ 2\Omega_{12} = 9.30\,\Gamma)$

Initial state: $|\Psi(0)\rangle = |e_1\rangle \otimes |e_2\rangle$ (both atoms excited)

No initial entanglement

Initial state: $|\Psi(0)\rangle = |e_1\rangle \otimes |e_2\rangle$ (both atoms excited)

No initial entanglement



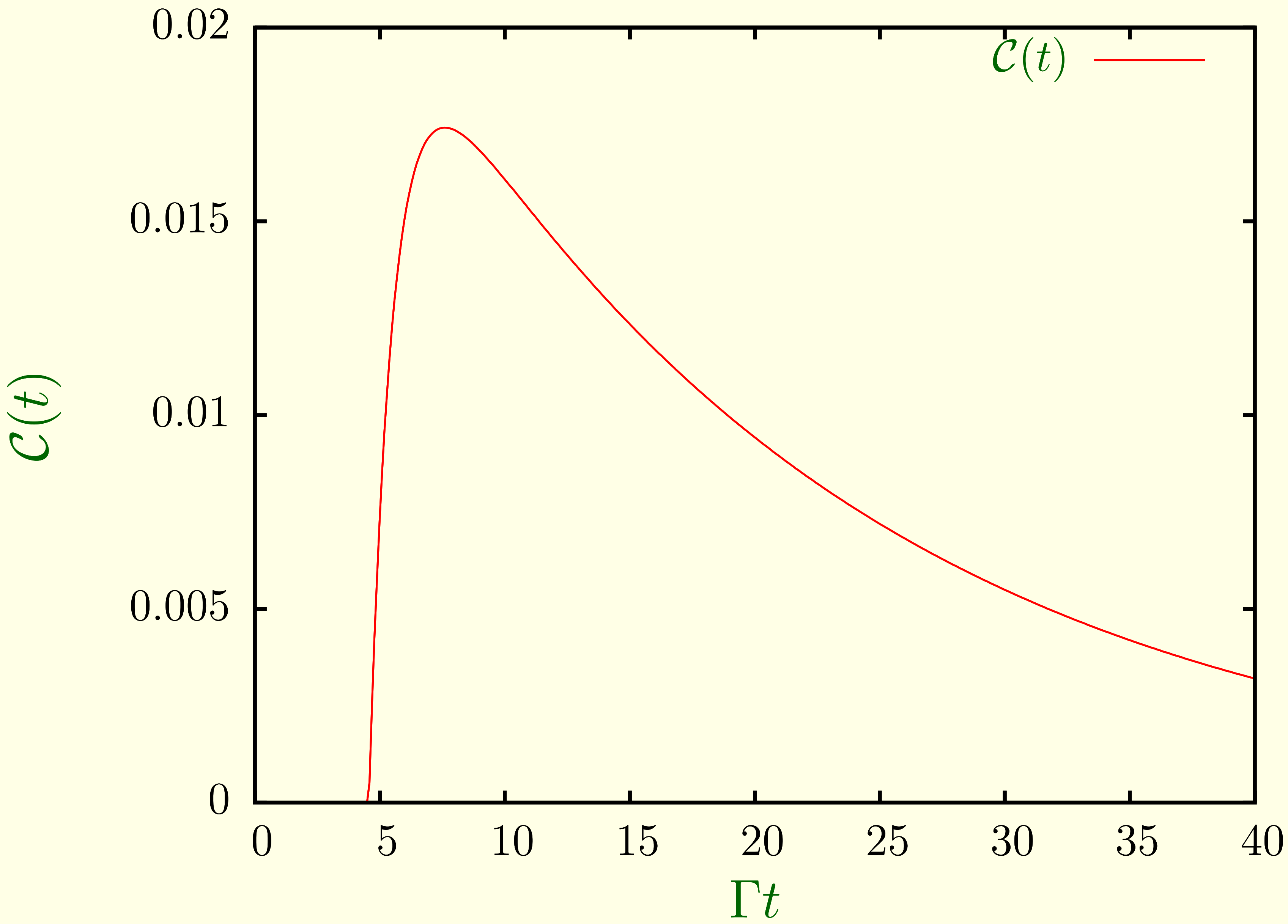
Sudden birth of entanglement: $\mathcal{C}(t > t_b) > 0$

Concurrence:

$$\mathcal{C}(t) = \max\left\{0, \mathsf{C}_2(t)\right\}$$

$$\mathsf{C}_2(t) = \left| \frac{\Gamma + \Gamma_{12}}{\Gamma - \Gamma_{12}} \left(\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t} \right) - \frac{\Gamma - \Gamma_{12}}{\Gamma + \Gamma_{12}} \left(\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t} \right) \right| - 2\mathrm{e}^{-\Gamma t}\sqrt{\rho_{gg}}$$

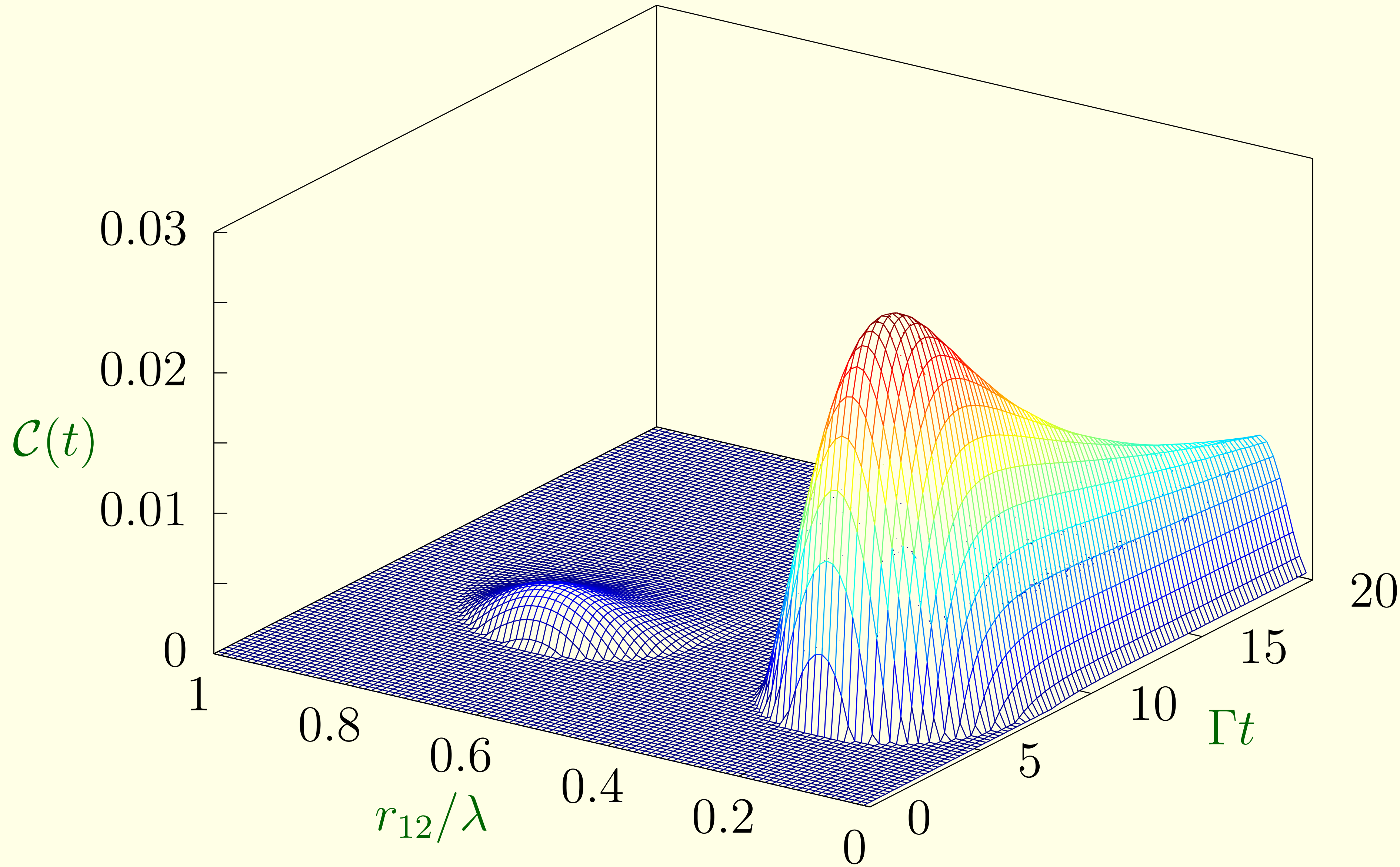
$$\rho_{gg}(t) = 1 - \left[\frac{\Gamma + \Gamma_{12}}{\Gamma - \Gamma_{12}} \left(\mathrm{e}^{-(\Gamma+\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t} \right) + \frac{\Gamma - \Gamma_{12}}{\Gamma + \Gamma_{12}} \left(\mathrm{e}^{-(\Gamma-\Gamma_{12})t} - \mathrm{e}^{-2\Gamma t} \right) + \mathrm{e}^{-2\Gamma t} \right]$$



Concurrence $\mathcal{C}(t)$ for $\rho_{ee}(0) = 1$;

$(r_{12} = \lambda/12$, $\Gamma_{12} = 0.95\,\Gamma)$

Z. Ficek, R. Tanaś, Phys. Rev A **77**, 054301 (2008)



Dynamics of entanglement in a dissipative environment. . .

Dynamics of entanglement in a dissipative environment. . .

. . . depends strongly on the initial conditions.

Dynamics of entanglement in a dissipative environment. . .

. . . depends strongly on the initial conditions.

Not only sudden death. . .

Dynamics of entanglement in a dissipative environment...

...depends strongly on the initial conditions.

Not only sudden death...

...but also sudden birth and revivals...

Dynamics of entanglement in a dissipative environment. . .

. . . depends strongly on the initial conditions.

Not only sudden death. . .

. . . but also sudden birth and revivals. . .

. . . can be observed. . .

Dynamics of entanglement in a dissipative environment...

...depends strongly on the initial conditions.

Not only sudden death...

...but also sudden birth and revivals...

...can be observed...

...when two atoms behave collectively!


Thank you!