Bohmian Mechanics:
A Trajectory Picture of Quantum Mechanics

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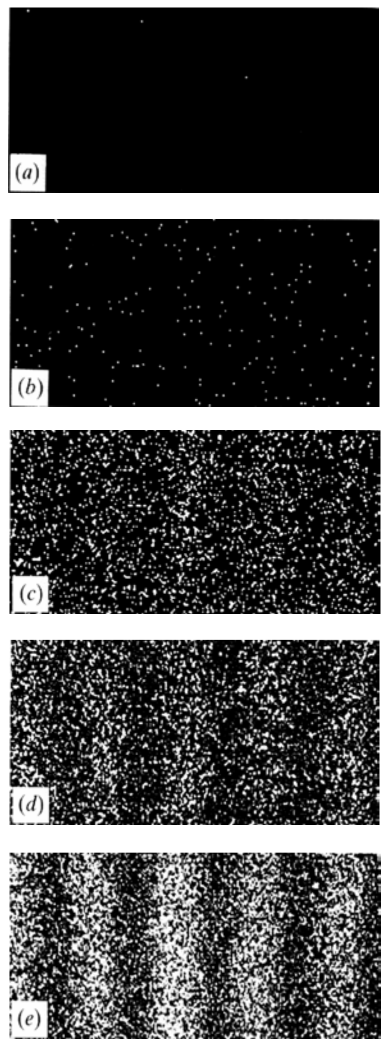
Why trajectory pictures of quantum mechanics?

Fundamental pictures of quantum mechanics:

- **Heisenberg (1925)** ⇒ Operators (“black box”)
- **Schödinger (1926)** ⇒ Deterministic wave fields
- **Feynman (1948)** ⇒ Classical-like paths and waves

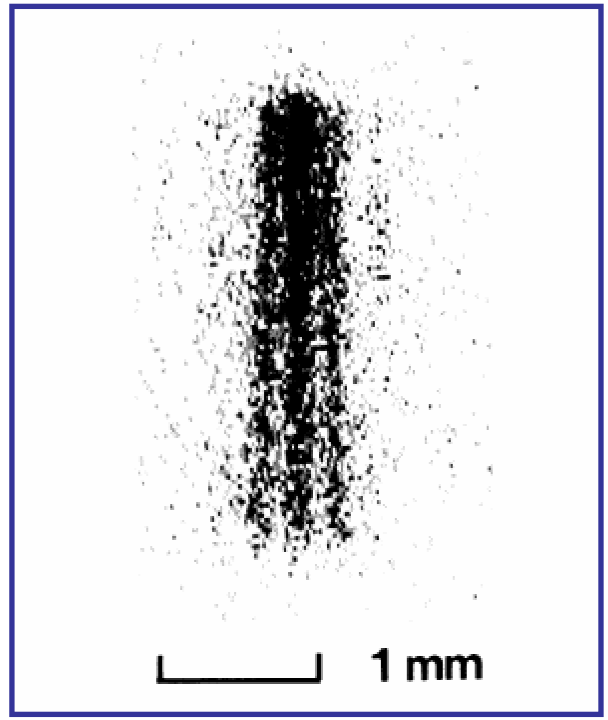
Quantum system = wave

Quantum systems: Waves or particles?



Demonstration of single-electron buildup of an interference pattern

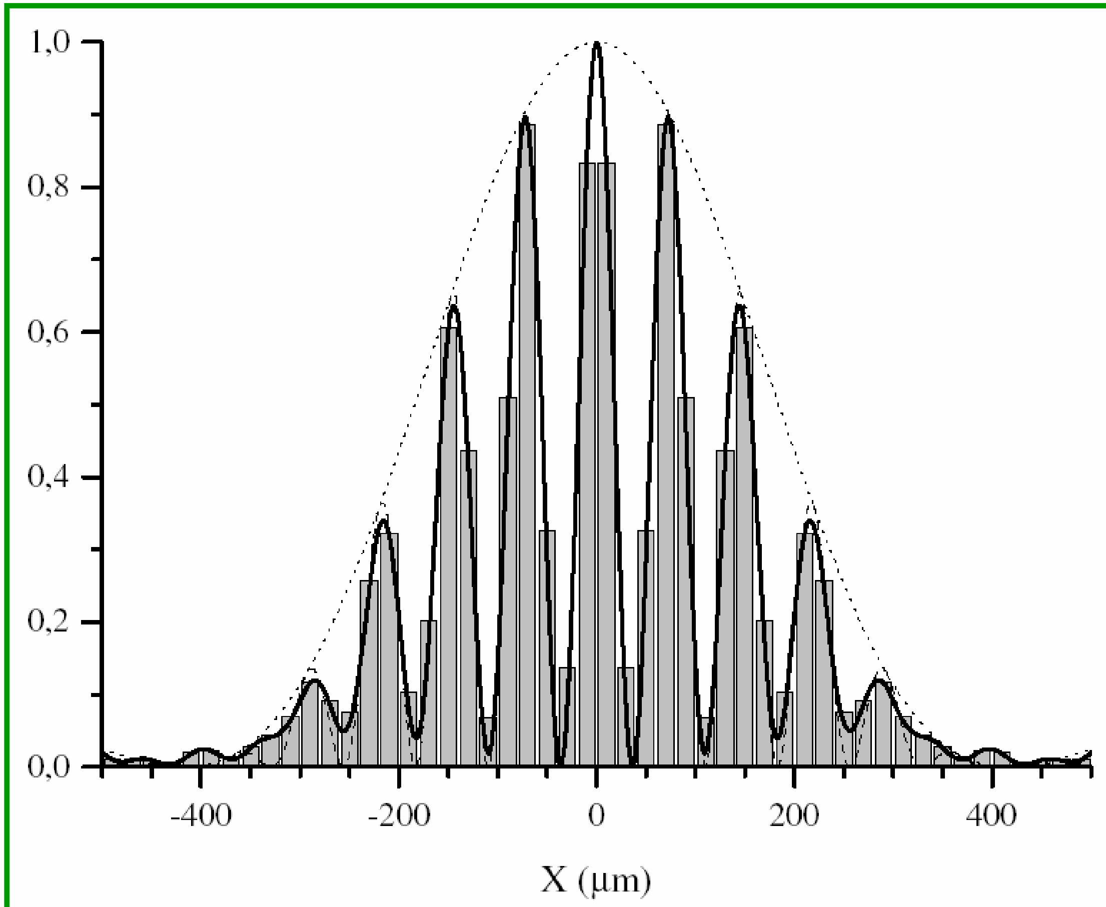
Tonomura, Endo, Matsuda, Kawasaki and Ezawa, *Am. J. Phys.* **57**, 117 (1989).



Double-slit interference with ultracold metastable neon atoms

Shimizu, Shimizu and Takuma, *Phys. Rev A* **46** R17 (1992).

Quantum systems: Waves or particles?



Quantum systems: Waves or particles?

Particle distributions behave as waves …

… but individual particles behave as individual point-like particles!

Explaining both behaviors within the same theoretical framework is precisely the reason why trajectory pictures of quantum mechanics are needed or desirable

What's Bohmian mechanics?

1) The evolution of the wave function, $\Psi(\vec{r},t)$, is given by the time-dependent Schrödinger equation.

2) The particle momentum is determined by Jacobi's law, $\vec{p} = \nabla S$, obtaining the Bohmian trajectories by integrating this law of motion.

3) There is no any prediction or control a priori over the particle initial position, but rather some statistical information about it given by the probability density, $\rho(\vec{r}) = \left|\Psi(\vec{r})\right|^2$.

What's Bohmian mechanics?

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\Psi+V\Psi$$

$$\Psi=R\,e^{iS\hbar}$$

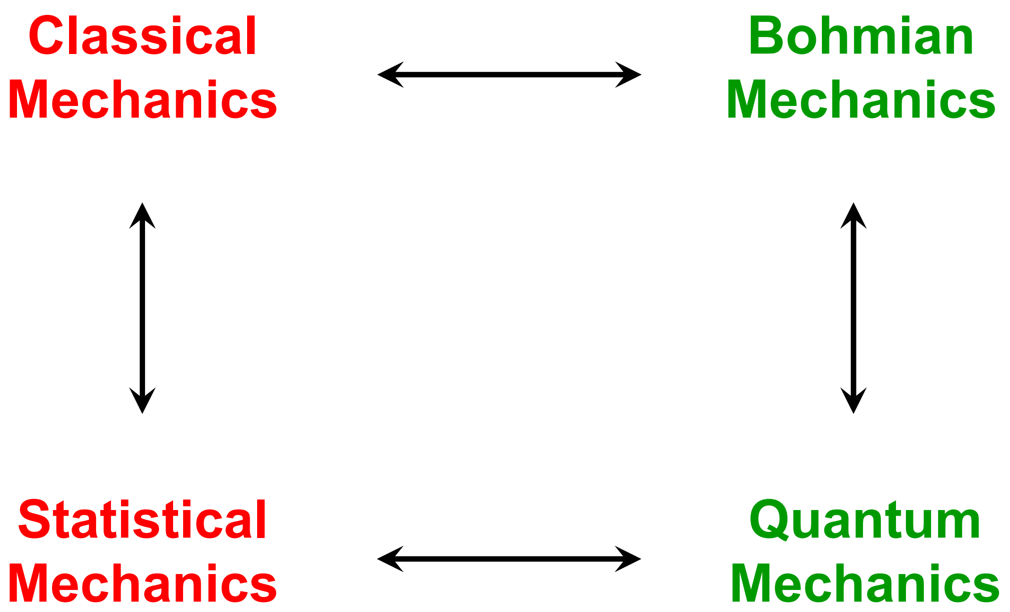
$$\longrightarrow$$

$$\begin{cases}\dfrac{\partial S}{\partial t}+\dfrac{(\nabla S)^2}{2m}+V-\dfrac{\hbar^2}{2m}\dfrac{\nabla^2 R}{R}=0\\[2ex]\dfrac{\partial R^2}{\partial t}+\nabla\cdot\left(R^2\dfrac{\nabla S}{m}\right)=0\end{cases}$$

$$\Downarrow$$

$$\begin{cases}i\hbar\dfrac{\partial\Psi}{\partial t}=-\dfrac{\hbar^2}{2m}\nabla^2\Psi+V\Psi\\[2ex]\vec{p}=\nabla S\end{cases}$$

Why Bohmian mechanics?



Bohmian mechanics: The state-of-the-art

➢ Mathematical foundations and implied philosophy

P.R. Holland – The Quantum Theory of Motion (1993)

D. Dürr – Bohmsche Mechanik (2001)

➢ Development of trajectory-based algorithms

R.E. Wyatt – Quantum Dynamics with Trajectories (2005)

➢ Reinterpretation of quantum phenomena

A.S. Sanz, S. Miret-Artés – Trajectory Pictures of Quantum Mechanics (2009?)

Applications

- Diffraction by slit gratings
- Atom-surface scattering
- Fractal Bohmian mechanics
- Beam interference and interferometry

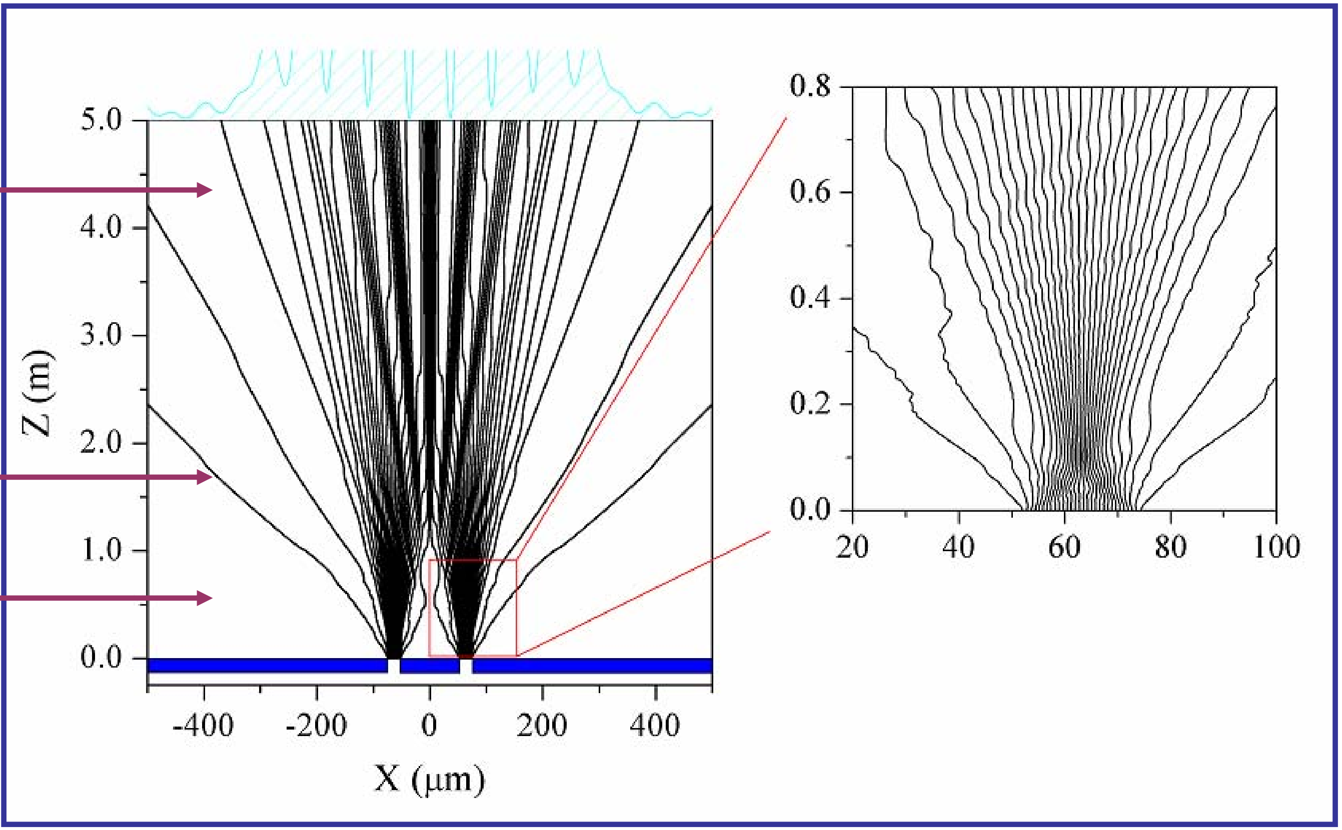
Applications: Two-slit diffraction

1.- Dynamical characterization of optical/quantum regions

Fraunhofer

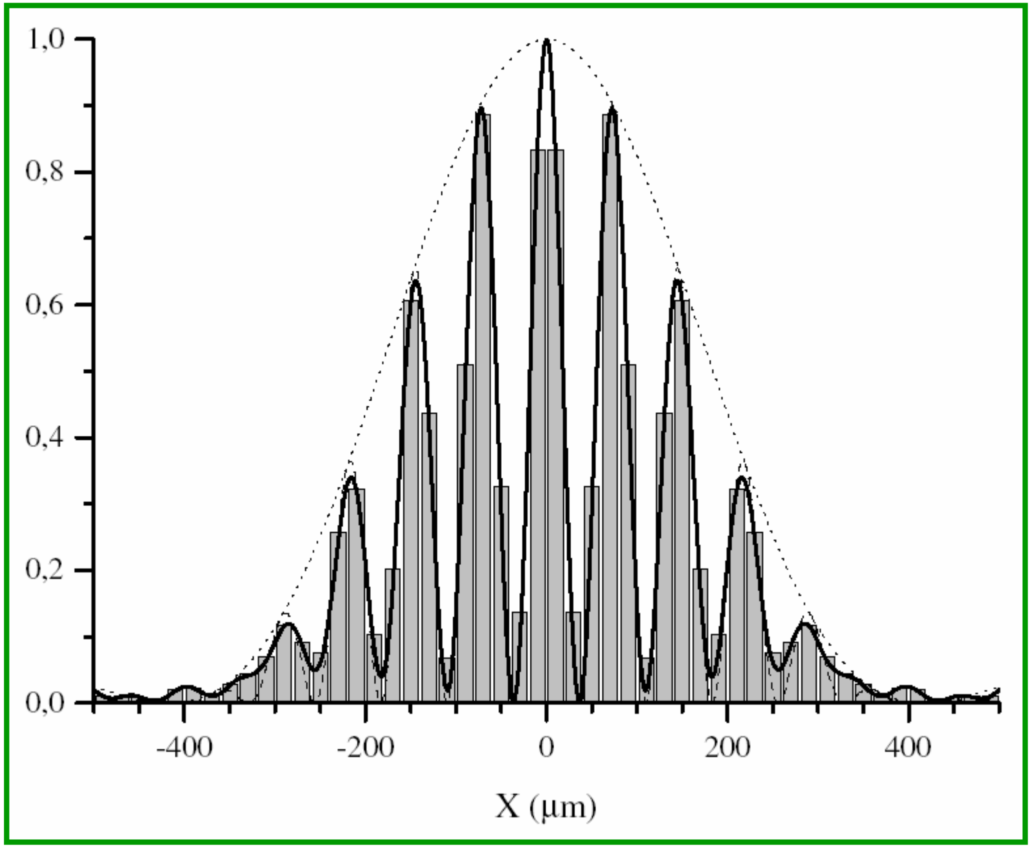
Transition

Fresnel



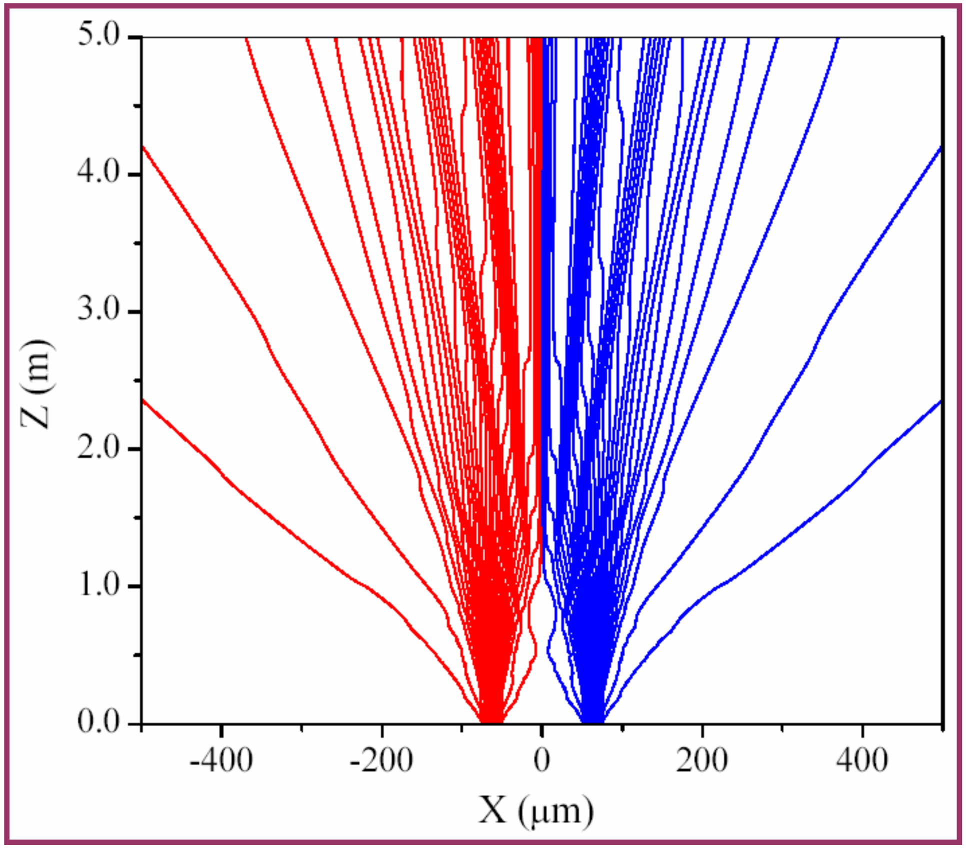
Applications: Two-slit diffraction

2.- Reproduction of phenomena and/or effects as in the experiment

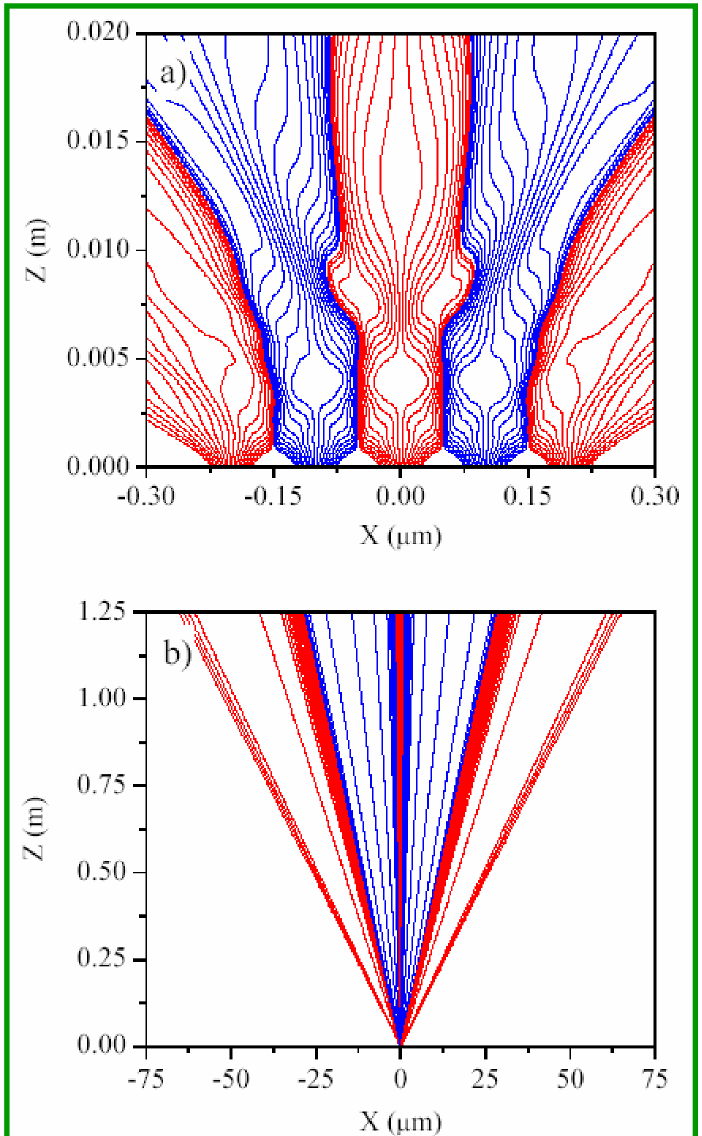


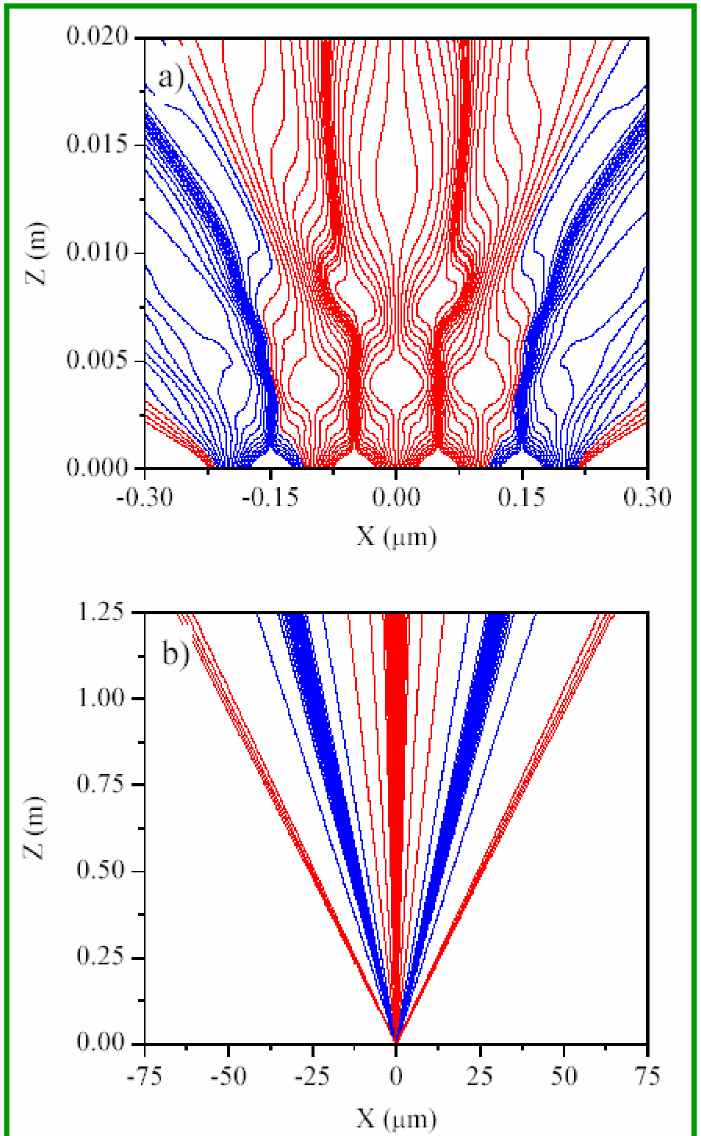
Applications: Two-slit diffraction

3.- Contextuality: Particles only cross one slit; the wave, both

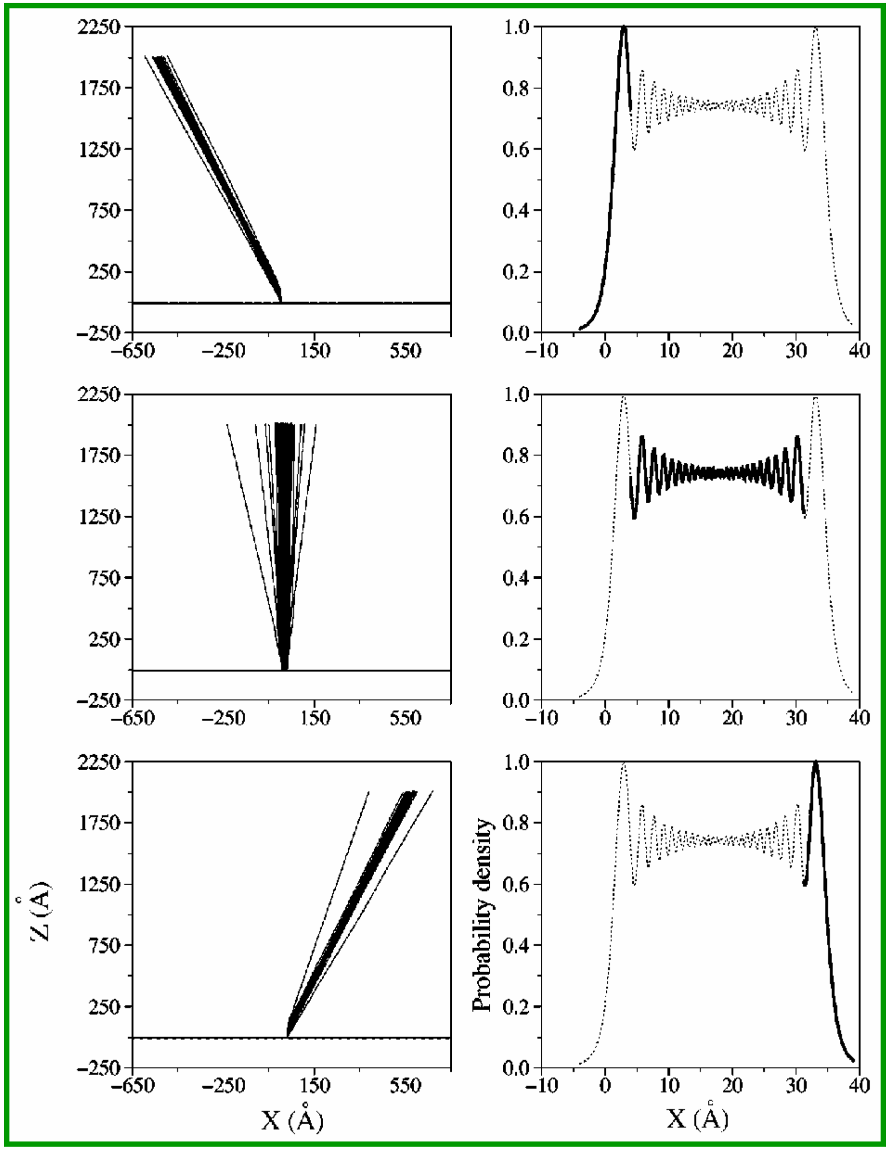


Applications: Multi-slit diffraction

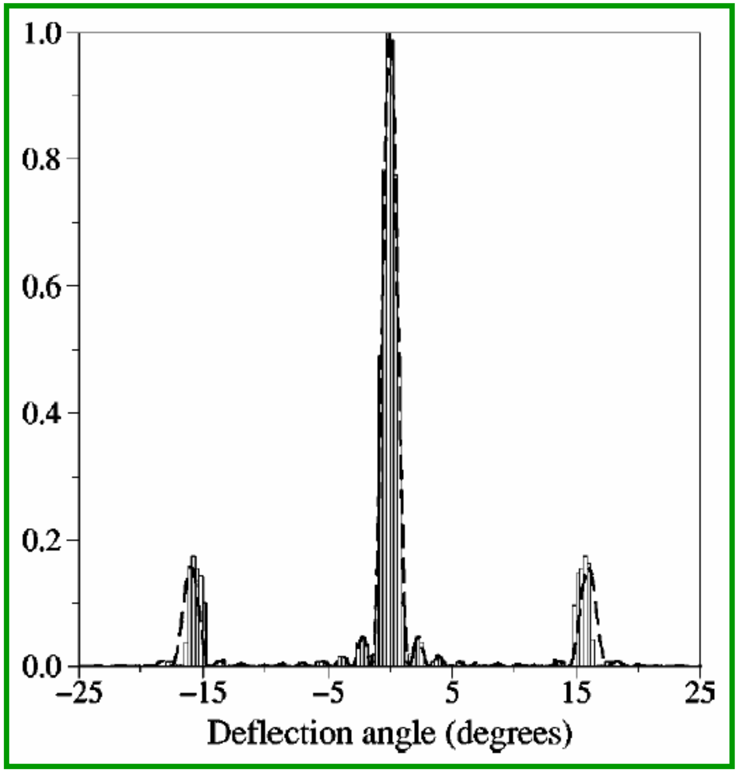




Applications: He-Cu elastic scattering



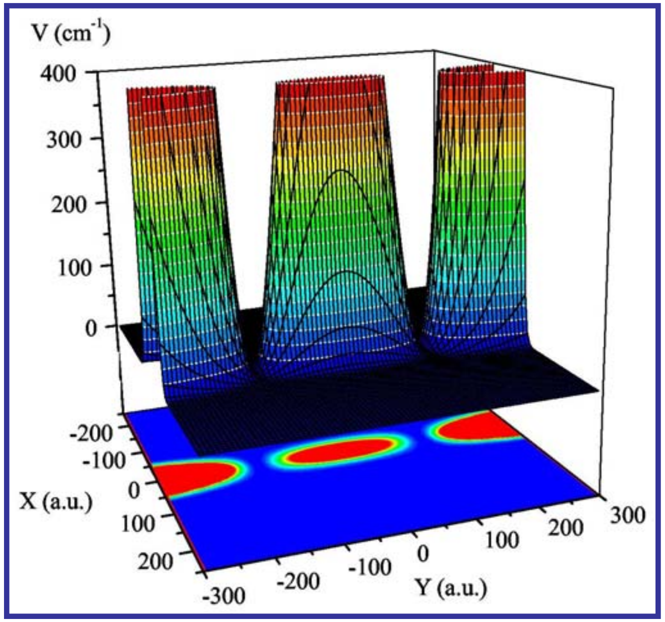
4.- Initial and final regions of configuration (position) space can be unambiguously related

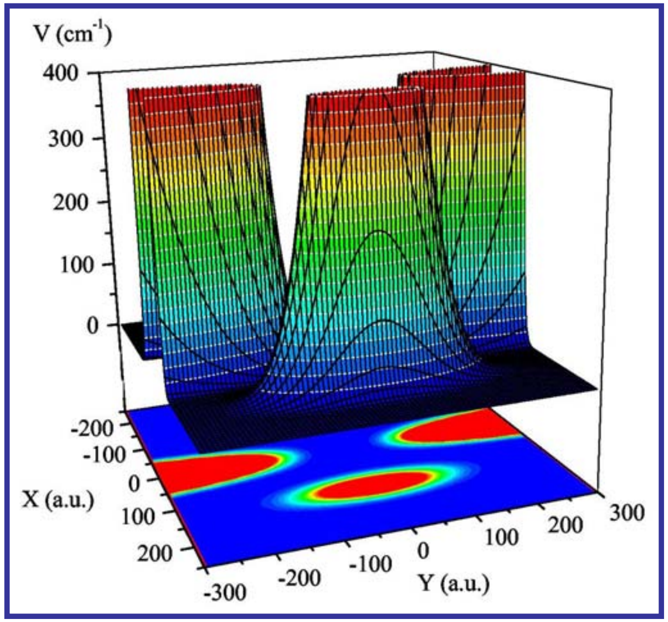


Applications: Two-slit diffraction

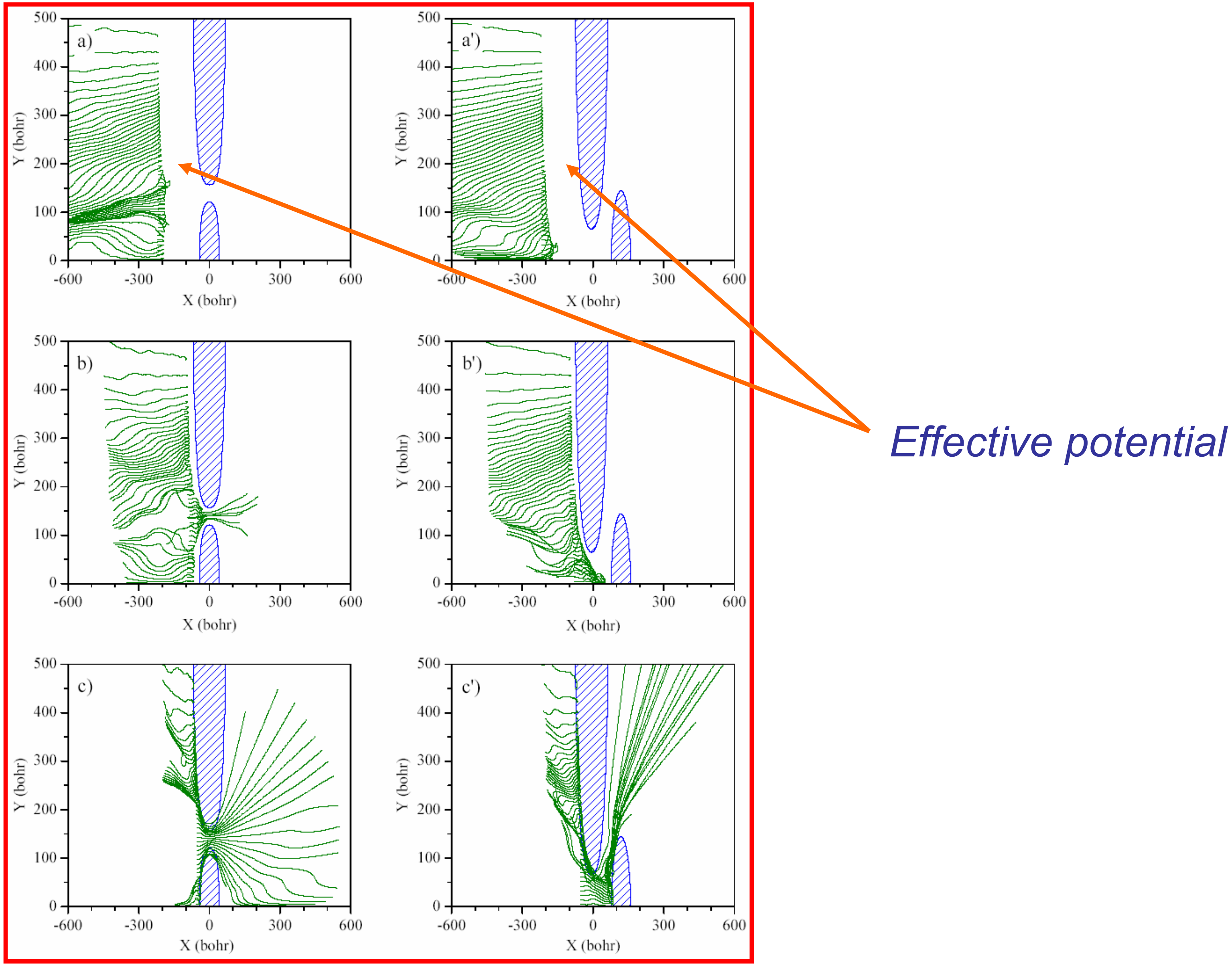
5.- Quantum particles are affected by a sort of quantum pressure

Soft (realistic) two-slit potential

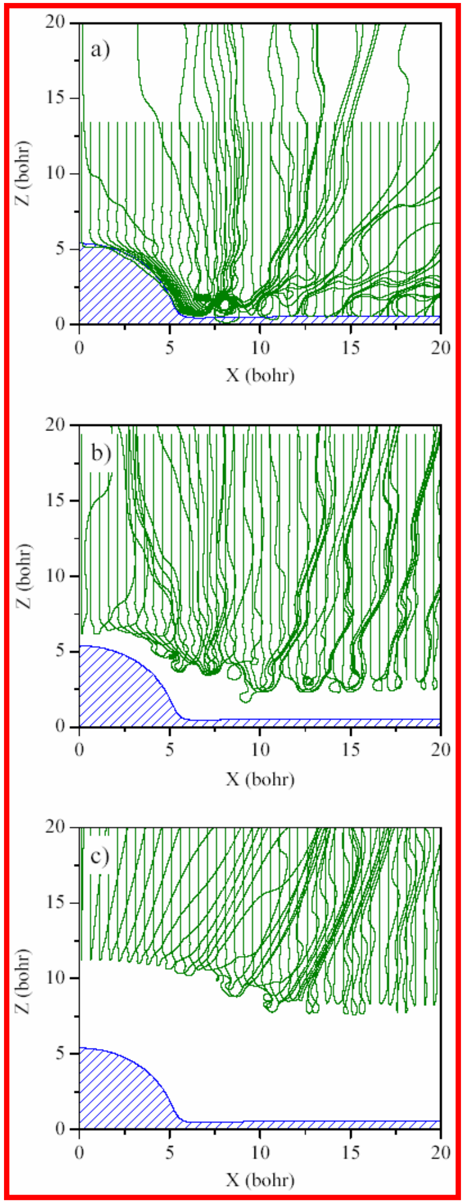




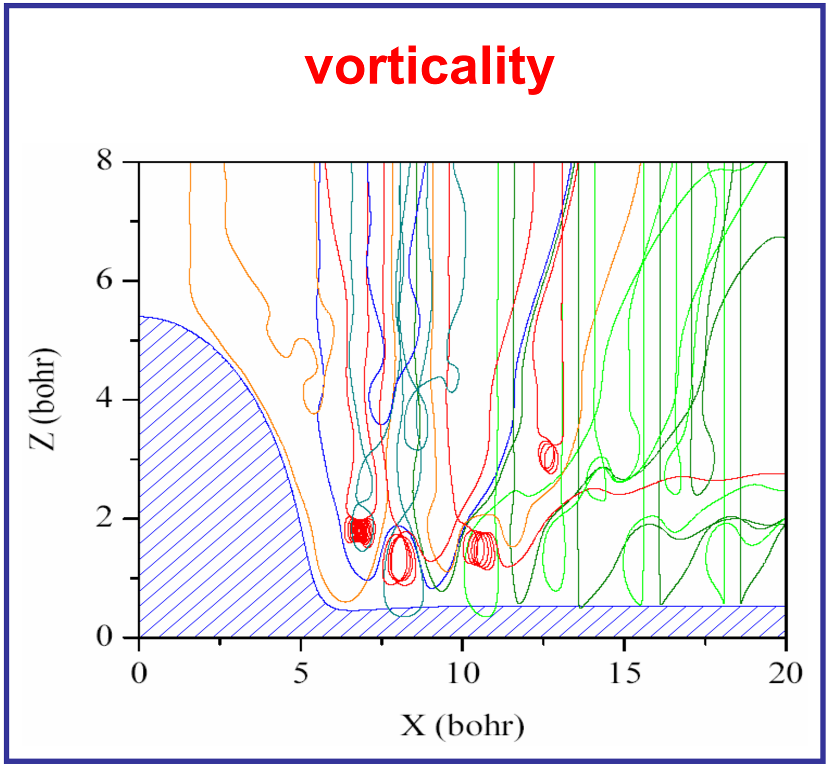
Applications: Two-slit diffraction



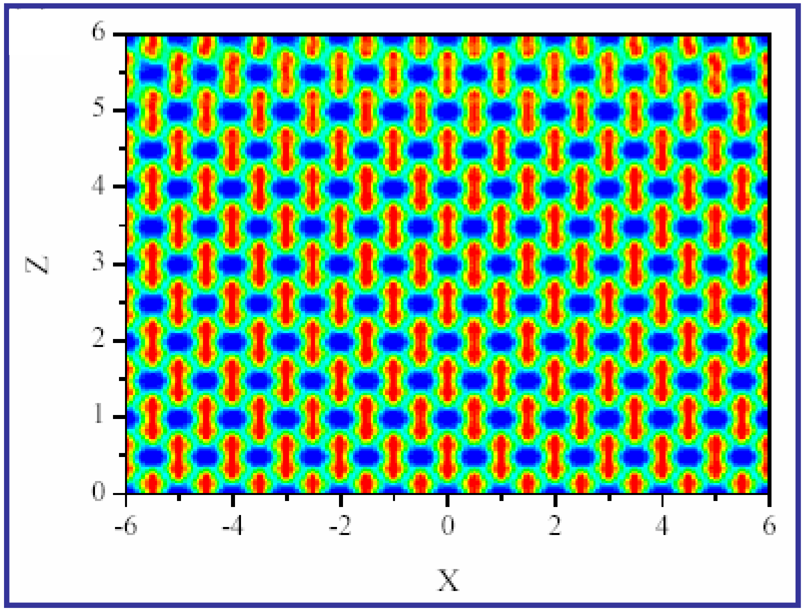
Applications: He-CO/Cu elastic scattering



6.- Detection of quantum vortices



Applications: The Talbot effect



7.- Causal explanation and characterization of the *Talbot effect*

Periodicity in x: d

Periodicity in z: $2z_T = \frac{2d^2}{\lambda}$

Talbot structure or quantum carpet

Talbot distance

Applications: The Talbot effect

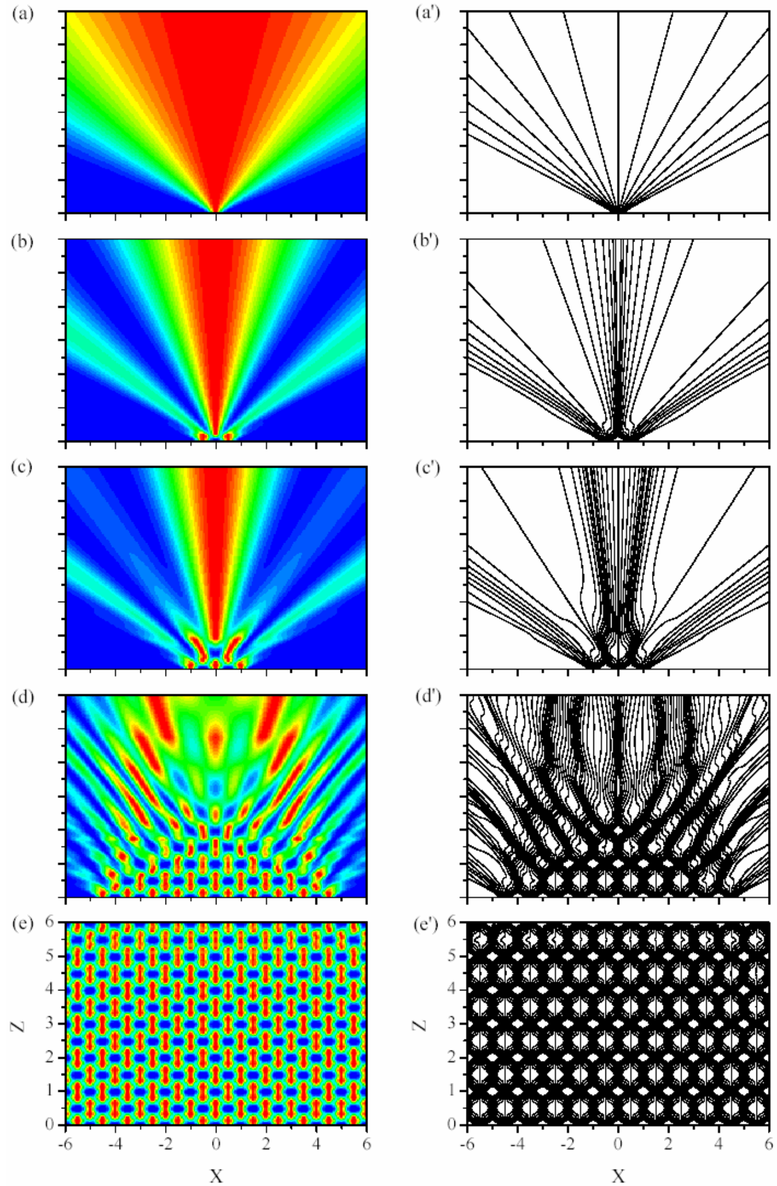
N = 1

N = 2

N = 3

N = 10

N = 50



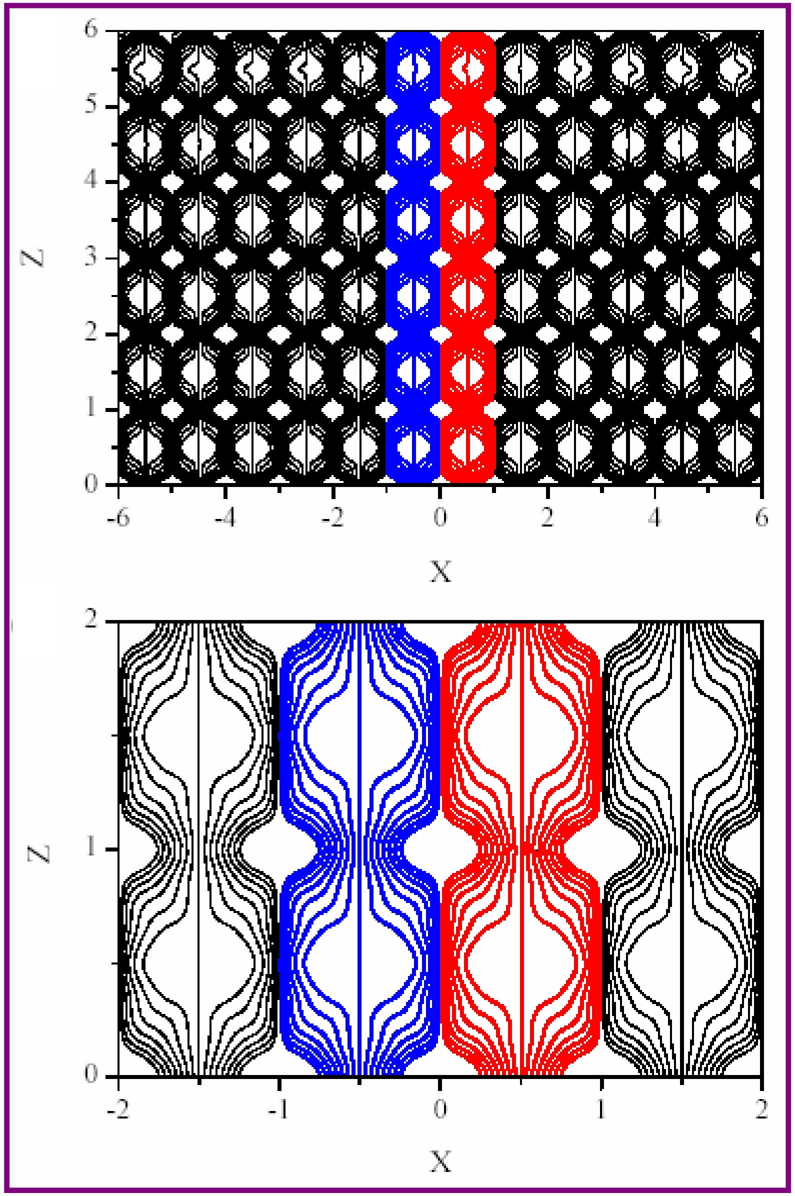
Talbot structure

Applications: The Talbot effect

channel structure

↓

Multimode cavities



Applications: The classical limit

8.- Causal characterization of the classical limit

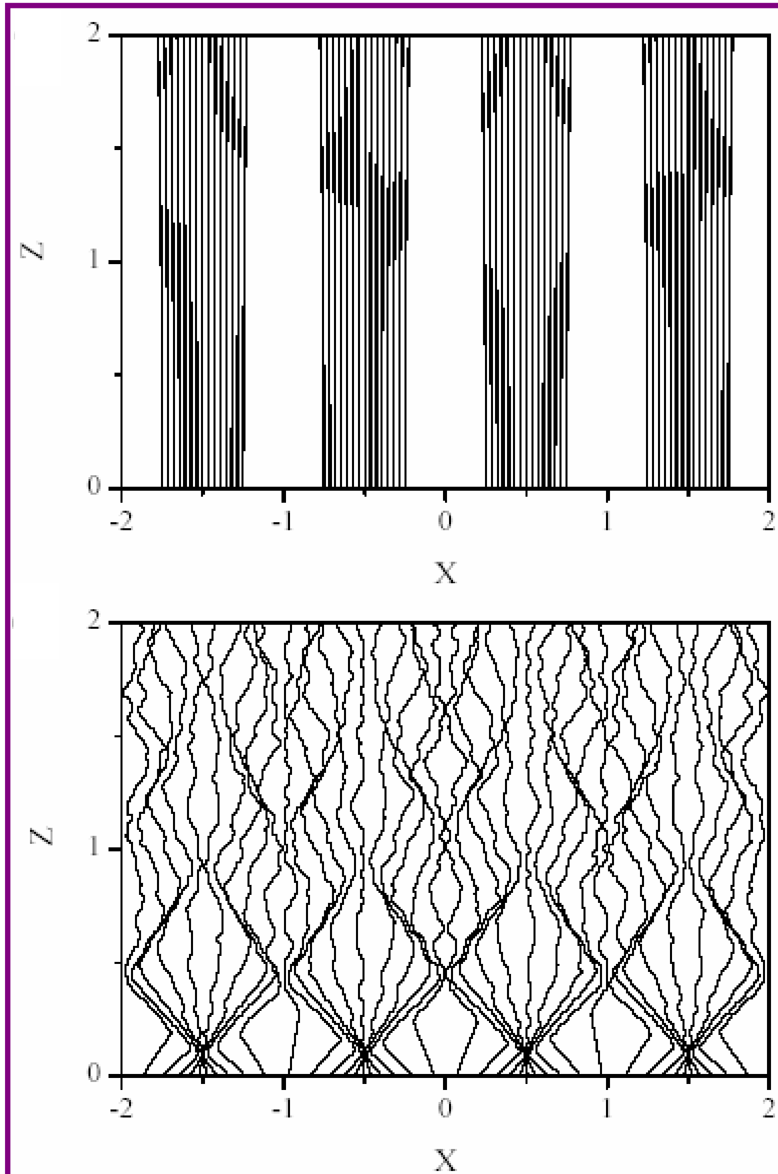
slits

- **no diffraction** → **direct transmission**

surfaces

- **rainbow-like trajectory patterns**

$$\rho_q^{av}(x,z;t) \approx \rho_{cl}(x,z;t)$$



Applications: Fractal Bohmian mechanics

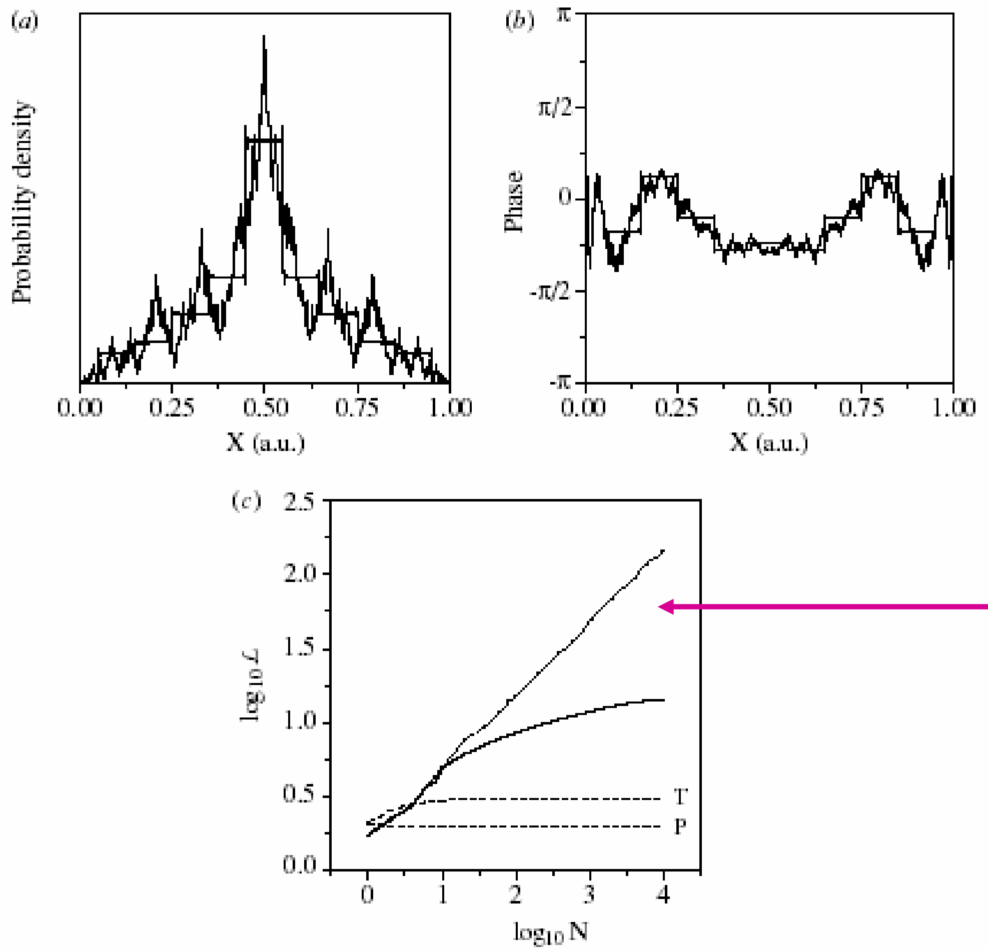


Figure 1. Probability density (*a*) and phase (*b*) associated with a highly delocalized particle in a box at $t = T/\sqrt{2}$ (thin solid line) and $t = 0.7T$ (thick solid line). (*c*) Measure of the fractal dimension of the probability densities displayed in part (*a*). To compare, measures of the fractal dimension of initial probability densities associated with triangular (T) and parabolic (P) wavefunctions are also shown.

fractal behavior

Applications: Fractal Bohmian mechanics

9.- Generalization of Bohmian mechanics to deal with *fractal* quantum states

Fractal quantum dynamics:

$$\left.\begin{aligned} &\Psi_t(x;N) = \sum_{n=1}^{N} c_n \xi_n(x) e^{-iE_n t/\hbar} \\ &\dot{x}_N(t) = \frac{\hbar}{m} \mathrm{Im}\left\{ \Psi_t^{-1}(x;N) \frac{\partial \Psi_t(x;N)}{\partial x} \right\} \end{aligned}\right\} \longrightarrow \left\{\begin{aligned} &\Psi_t(x) \equiv \lim_{N\to\infty} \Psi_t(x;N) \\ &x_t \equiv \lim_{N\to\infty} x_N(t) \end{aligned}\right.$$

Applications: Fractal Bohmian mechanics

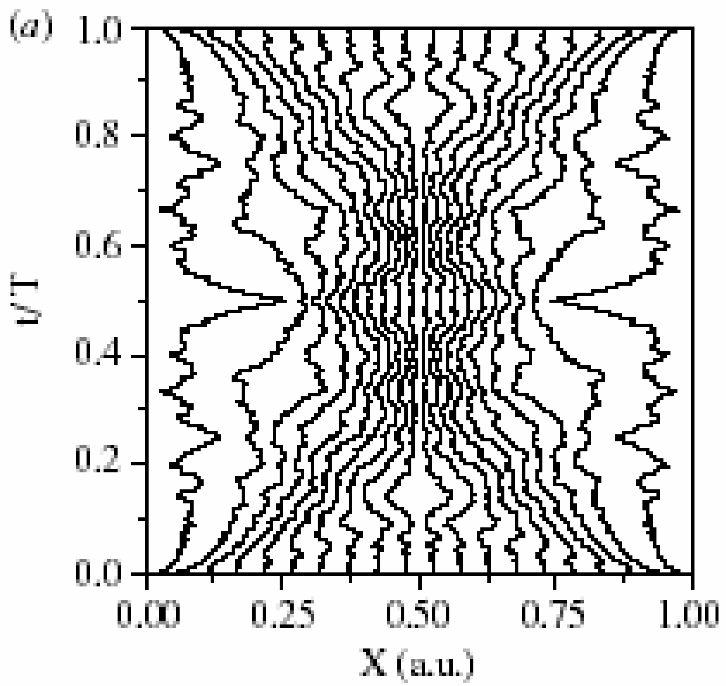
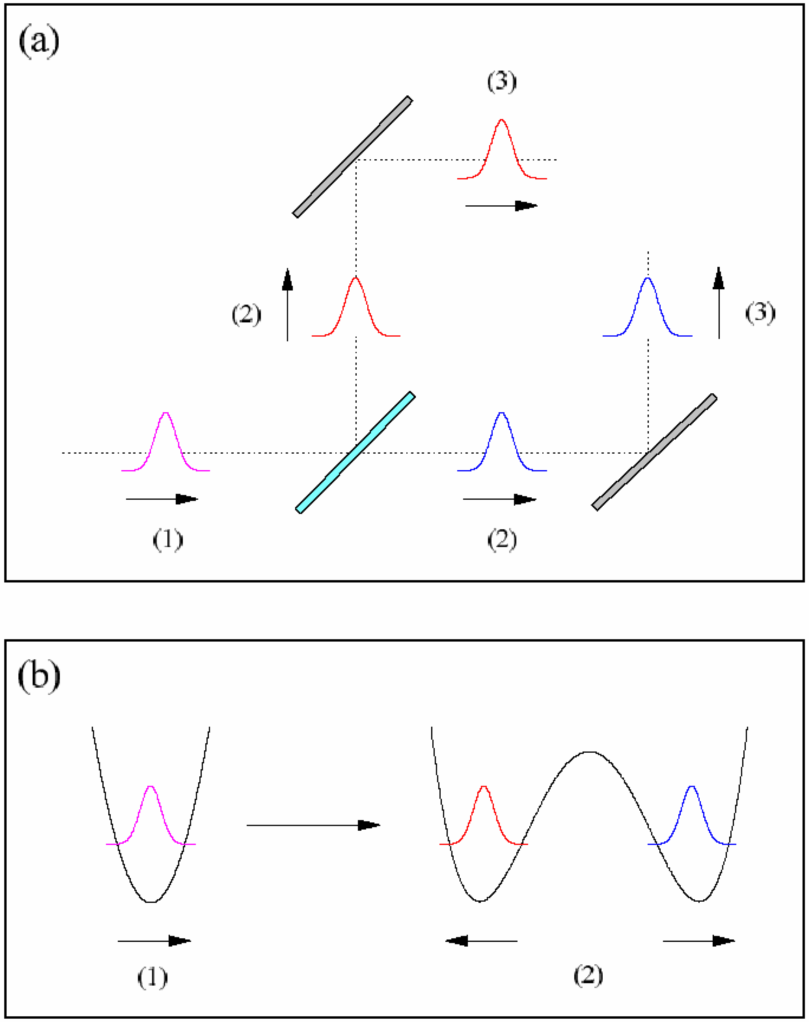
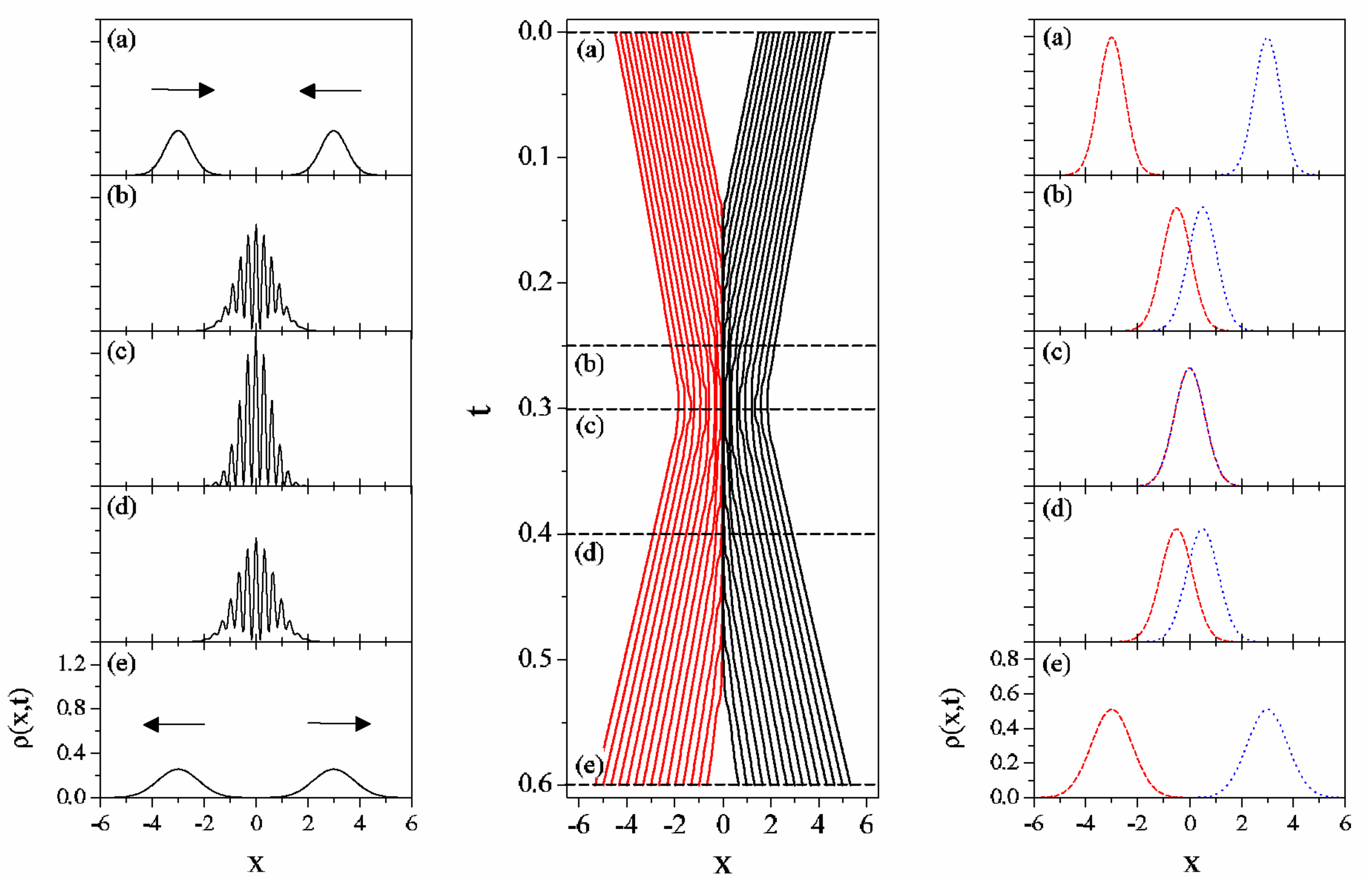


Figure 2. (*a*) QF-trajectories associated with a highly delocalized particle in a box. (*b*) Measure of the fractal dimension of a sample of QF-trajectories with initial positions: $x_0 = 0.01$ (■), $x_0 = 0.1$ (●), $x_0 = 0.4$ (▲), $x_0 = 0.49$ (□), $x_0 = 0.499$ (○), and $x_0 = 0.5$ (△).

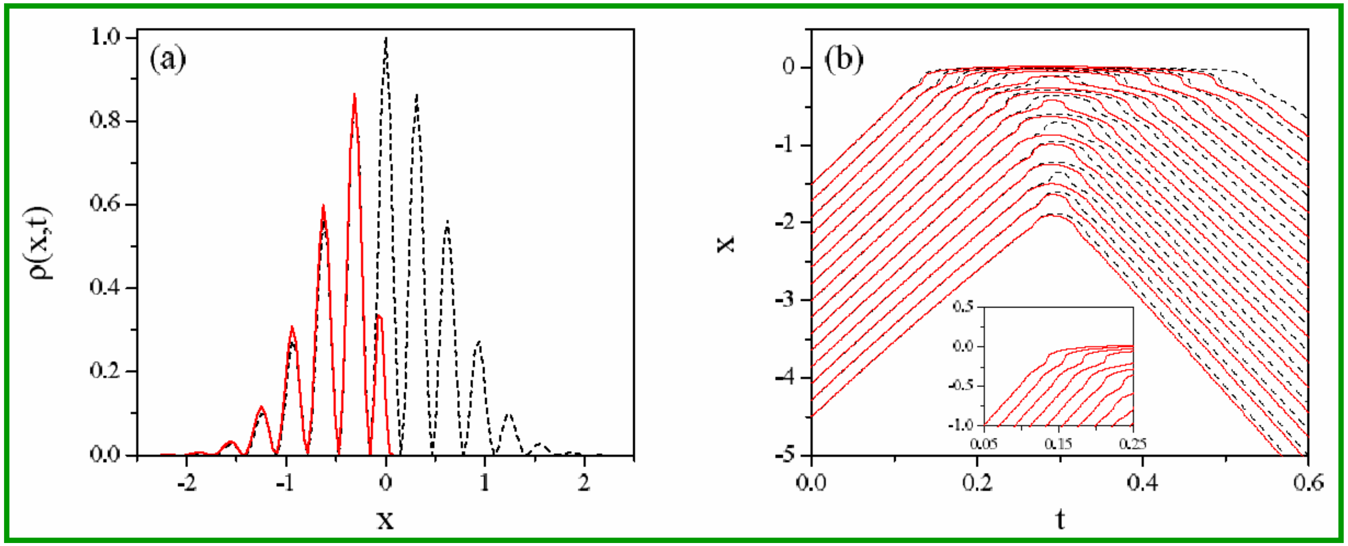
Applications: Interference and interferometry

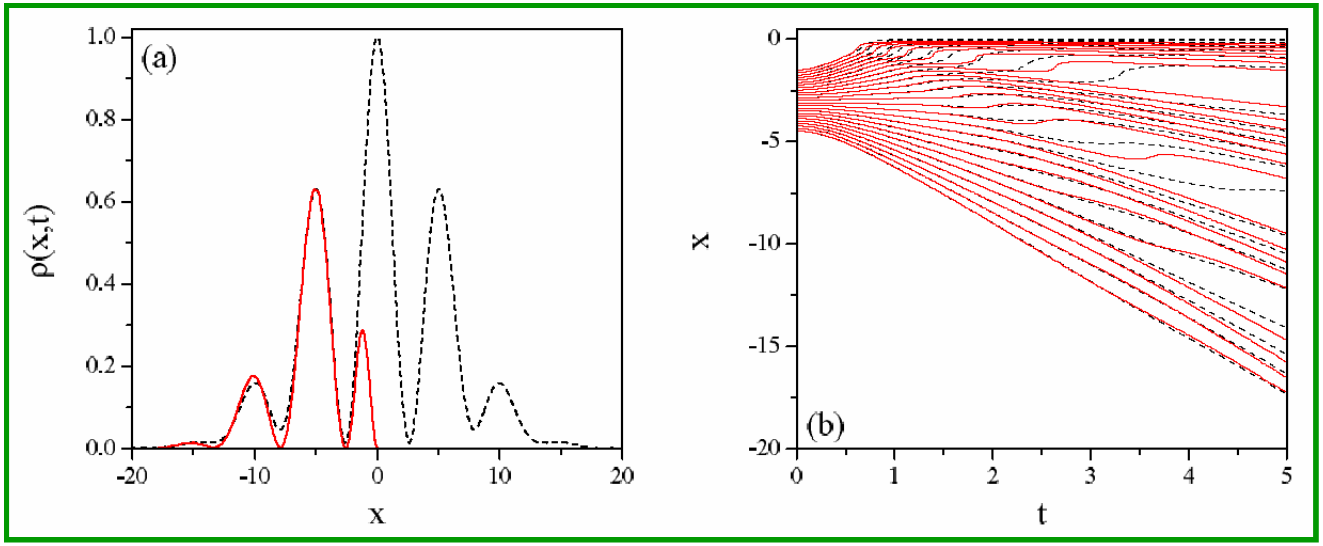


Applications: Interference and interferometry



Applications: Interference and interferometry





Conclusions

Bohmian mechanics provides a robust and consistent framework to analyze and understand the dynamical behavior of quantum systems, which allows to treat particles as in classical mechanics (i.e., as individual entities) and, at the same time, to observe the well-known wave-like behaviors characteristic of the standard version of quantum mechanics.

In other words, Bohmian mechanics can be an important tool to create the quantum intuition necessary to think the quantum world.

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