Entanglement in the adiabatic limit of cavity QED with pairs of atoms

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CEWQO 2008

Outline

Motivation

- Model the sequential passage of atoms through a cavity.
- Adiabatic limit for cavity QED.
- Potential formation of entanglement.
- Quantum Computing applications

Sequential passage of atoms

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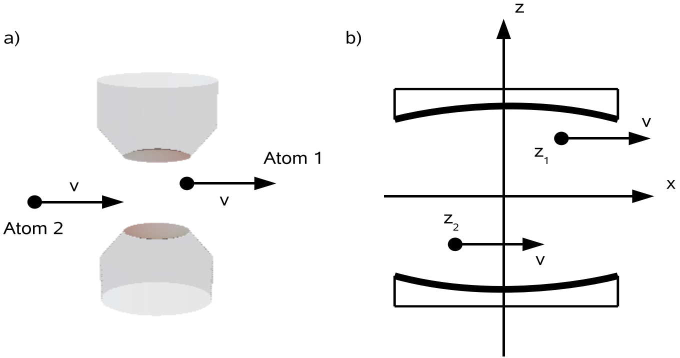

Time-dependent coupling profiles

- Utilising the couplings with time-dependent pulses $\eta_j(\tau)$

$$H(\tau) = \frac{1}{2}\sum_{i=1,2}\Delta_i\sigma_z^i + \sum_{i=1,2}\eta_i(\tau)\left(\sigma_-^i a^\dagger + \sigma_+^i a\right)$$

- Center of mass motion is classical:

$$v_i \gg (n+1)^{1/4}\sqrt{\frac{2\hbar g_i}{m}}$$

- For open cavities with curved mirrors

$$\eta_1(\tau) = g_1 e^{-(\tau+\delta)^2} \qquad \eta_2(\tau) = g_2 e^{-(\tau-\delta)^2}$$

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Time-dependent coupling profiles

Our model incorporates:

- Time delays between the atoms: δ.

$$\delta = \frac{v\Delta t}{w_0}$$

- Asymmetries in the coupling profiles: $g_1 \neq g_2$.

$$g_i = g_0 \cos(kz_i) e^{-\frac{y_i^2}{w_0^2}}$$

- Frequency chirps: $\Delta_i = \Delta_i(\tau)$.

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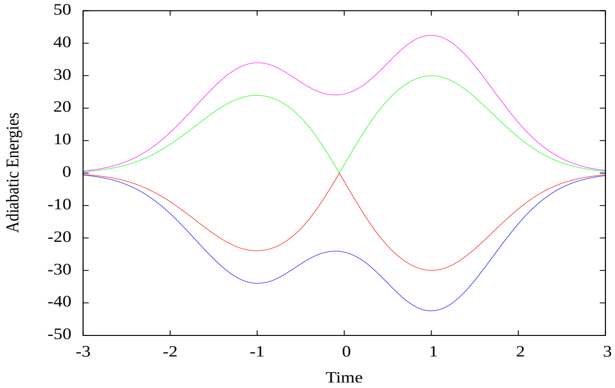
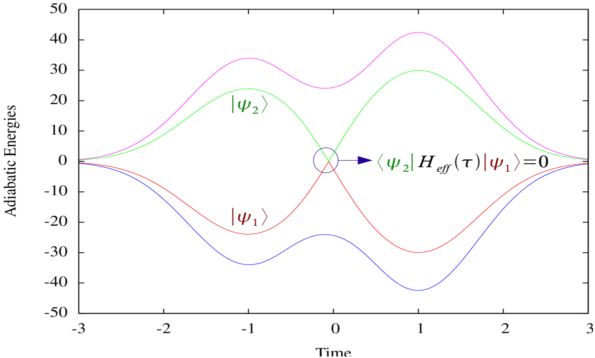
Energy crossing

- Temporal degeneracy between adiabatic states.



- The degenerate states do not interact $\Rightarrow$ Pure crossing.



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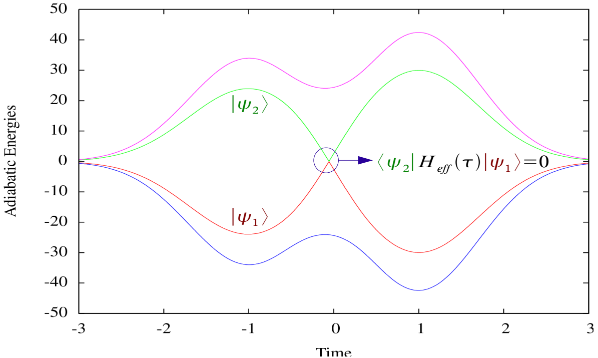

Adiabatic approximation

Adiabatic evolution:

- Atoms must coexist for a period of time:

$$\Delta t \approx f \frac{w_0}{v}, \qquad f \approx 1.0 - 1.25$$

- System must be in the strong coupling regime:

$$\frac{g_i w_0}{2v} \gg \alpha n^{\gamma}, \qquad \alpha \approx 0.2 - 0.3, \quad \gamma \approx 0.7 - 0.9$$

- Feasible with current experimental conditions (Raimond et. al. RMP 2001)

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Conditional entanglement

- The second atom entangles with the cavity mode.
- Entanglement:
  1. Conditional upon the initial state of the atom.
  2. Depends on two dynamical parameters.
- For symmetric interactions: $g_1 = g_2$
  1. One of the two parameters is zero.
  2. Coherent control via a single parameter.
  3. Robust control with Stark-chirps.
  4. Applications: Logic gates, state mapping, teleportation.
  5. Maximally entangled states.
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Equal couplings: $g_1 = g_2$

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$$|\psi_0\rangle = \frac{1}{2}|0\rangle \otimes (|g_1\rangle + |e_1\rangle) \otimes (|g_2\rangle + |e_2\rangle)$$

- Output: Tripartite entanglement.
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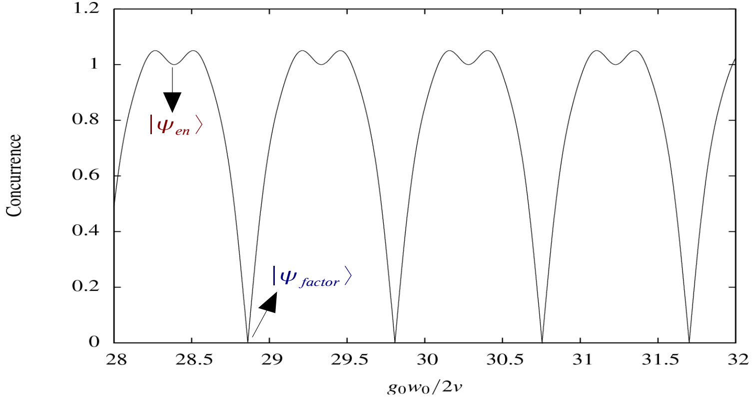
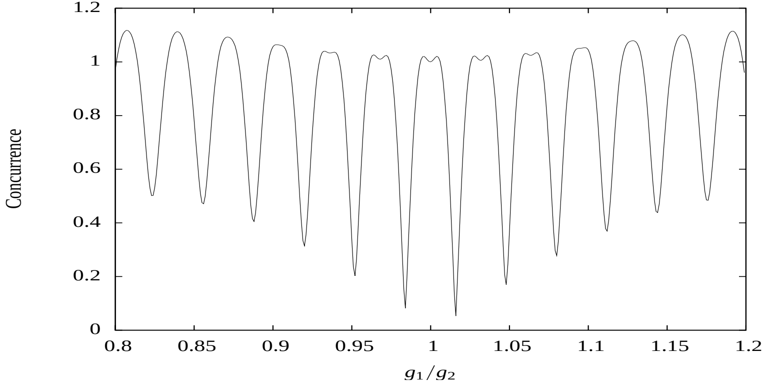
- Tripartite concurrence (Mintert et. al. Phys. Rep. 2005)

$$C_3(|\psi\rangle) = \sqrt{3 - \mathrm{tr}\rho_1^2 - \mathrm{tr}\rho_2^2 - \mathrm{tr}\rho_c^2}$$



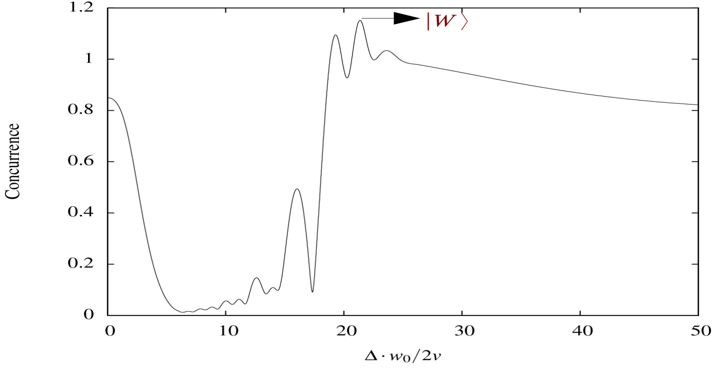
Unequal couplings: $g_1 \neq g_2$

- Tripartite entanglement.
- No factor states.



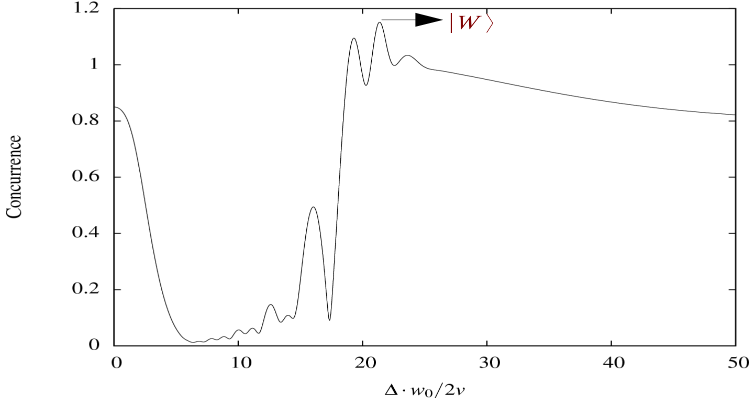
Off-resonant interactions

- Dynamics split into three regimes.
- $\Delta \ll g_0$ Similar to the resonant limit.

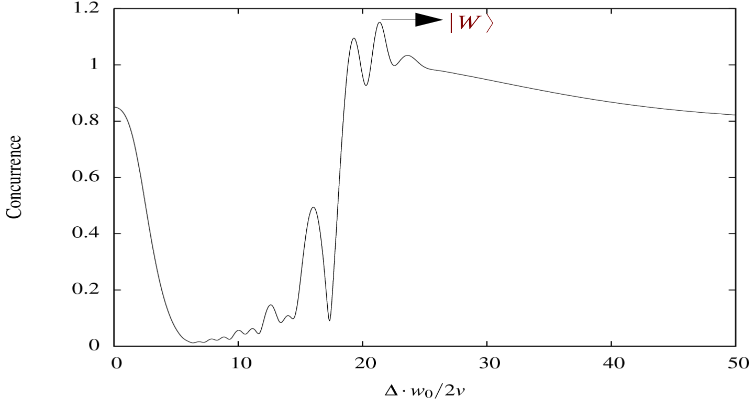


- $\Delta \geq g_0$ Tripartite entanglement.

$$|\psi\rangle \approx |W\rangle = \frac{1}{\sqrt{3}}\left(e^{i\theta_1}|0; e_1, e_2\rangle + e^{i\theta_2}|1; e_1, g_2\rangle + e^{i\theta_3}|1; g_1, e_2\rangle\right)$$

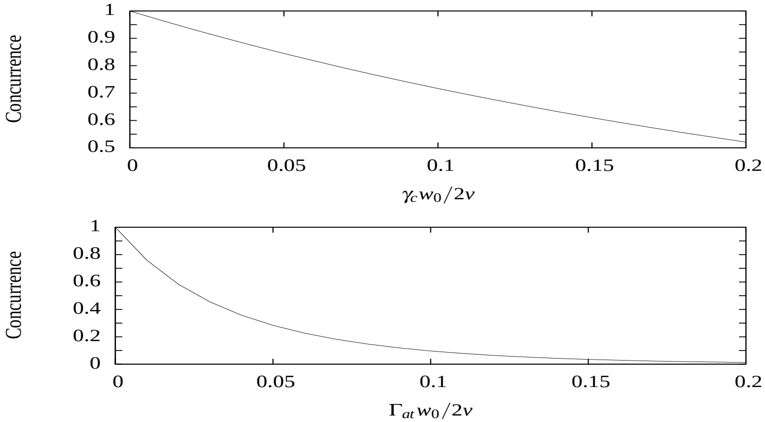


- $\Delta \gg g_0$ Atomic bipartite entanglement.



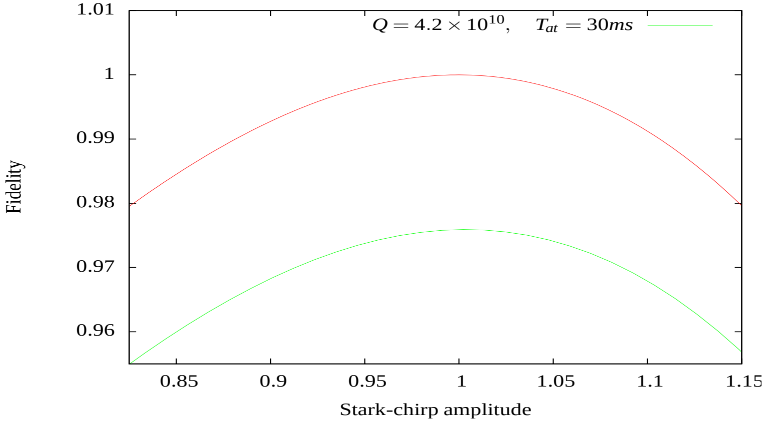
Decoherence and dissipation

- Entanglement decays.



- High Q micro-cavities (Kuhr et. al APL 2007)

$\Rightarrow$ Small dissipation



Conclusion

- Interesting physics: Energy crossing.
- Applications in Quantum Computing.
- Robust control of the system.
- Feasible with current experimental conditions.
- Potential use in Cavity QED with trapped ions?

References

- C. Lazarou and B. M. Garraway *Adiabatic entanglement in two-atom cavity QED.* Physical Review A **77** 023818 (2008)
- C. Lazarou and B. M. Garraway *Adiabatic cavity QED with pairs of atoms: Atomic entanglement and Quantum teleportation.* To be published European Physical Journal Special Topics (arxiv 0803.1479v1)